Games and Economic Behavior **34,** 123–137 (2001)
doi:10.1006/game.2000.0794, available online at http://www.idealibrary.com on IDEAL®

Zermelo and the Early History of Game Theory[1]

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Received September 3, 1997

In the modern literature on game theory there are several versions of what is known as Zermelo's theorem. It is shown that most of these modern statements of Zermelo's theorem bear only a partial relationship to what Zermelo really did. We also give a short survey and discussion of the closely related but almost unknown work by König (1927, *Acta Sci. Math. Szeged*, **3**, 121–130) and Kálmar (1928/29, *Acta Sci. Math. Szeged*, **4**, 65–85). Their papers extend and considerably generalize Zermelo's approach. A translation of Zermelo's paper is included in the Appendix. *Journal of Economic Literature* Classification Numbers: B19; C70; C72.

Key Words: history of game theory; chess; Zermelo.

1. INTRODUCTION

It is generally agreed that the first formal theorem in the theory of games was proved by E. Zermelo[3] in an article on Chess appearing in German in 1913 (Zermelo, 1913). In the modern literature on game theory there are

[1] We thank Graeme Guthrie, Harold Kuhn, Philip Meguire, Martin Osborne, Hartmut Stein, and an anonymous referee for their suggestions and comments.

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[3] Ernst Friedrich Ferdinand Zermelo (1871–1951), was a German mathematician. He studied mathematics, physics, and philosophy at Halle, Freiburg, and Berlin where he received his doctorate in 1894. He taught at Göttingen, Zürich, and Freiburg and is best known for his work on the axiom of choice and axiomatic set theory.

0899-8256/01 $35.00

many variant statements of this theorem. Some writers claim that Zermelo showed that Chess is determinate, e.g., Aumann (1989b, p. 1), Eichberger (1993, p. 9), or Hart (1992, p. 30): "In Chess, either White can force a win, or Black can force a win, or both sides can force a draw." Others state more general propositions under the heading of Zermelo's theorem, e.g., Mas-Colell *et al*. (1995, p. 272): "Every finite game of perfect information Γ_E has a pure strategy Nash equilibrium that can be derived by backward induction. Moreover, if no player has the same payoffs at any two terminal nodes, then there is a unique Nash equilibrium that can be derived in this manner." Dimand and Dimand (1996, p. 107) claim that Zermelo showed that White cannot lose: "[I]n a finite game, there exists a strategy whereby a first mover ... cannot lose, but it is not clear whether there is a strategy whereby the first mover can win." In addition many authors claim that Zermelo's method of proof was by backward induction, e.g., Binmore (1992, p. 32): "Zermelo used this method way back in 1912 to analyze Chess. It requires starting from the end of the game and then working backwards to its beginning. For this reason, the technique is sometimes called 'backwards induction'."

Despite a growing interest in the history of game theory (e.g., Aumann, 1989a, Dimand and Dimand, 1996, 1997, Kuhn, 1997, Leonard, 1995, and Weintraub, 1992), confusion, at least in the English language literature, as to the contribution made by Zermelo and some of the other early game theorists seems to prevail. This problem may be due in part to a language barrier. Many of the early papers in game theory were not written in English and have not been translated. For example, to the best of our knowledge, there is no English version of Zermelo (1913). The same holds for the lesser known but related work by König (1927).[4] A second paper related to that of Zermelo, by Kalmár (1928/29),[5] has recently been translated (see Dimand and Dimand, 1997).[6] The lack of an English translation may help to explain the apparent confusion in the modern literature as to what Zermelo's theorem states and the method of proof employed. It appears that there

[4]Dénes König (1884–1944), was a Hungarian mathematician, the son of the mathematician Julius König. He studied mathematics in Budapest and Göttingen and received his doctorate in 1907. He spent his whole career in Budapest, first as an assistant and later as a professor. Most of König's work was in the field of combinatorics and he wrote the first comprehensive treatise on graph theory, *Theorie der endlichen und unendlichen Graphen* (1936) ('Theory of Finite and Infinite Graphs').

[5]László Kalmár (1905–1976) was also a Hungarian mathematician. He studied mathematics and physics in Budapest. From 1930 until his death he worked at Szeged University, first as an assistant, later as a professor. His main research was in mathematical logic, computer science, and cybernetics.

[6]However, the translation of Kalmár's paper contains so many mistakes that it is almost impossible to understand what Kalmár did.