Japan and the World Economy
13 (2001) 299–302

Japan and the WORLD ECONOMY

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Some game theory anecdotes

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Received 31 March 2000; received in revised form 7 February 2001; accepted 25 February 2001

Abstract

After 1944, economists learned about Johnny von Neumann's two-person zero-sum game theory and Nash's non-cooperative game solution. The present discussion links the Nash analysis back to much earlier 1838 Cournot duopoly theory. And it links the von Neumann breakthrough to earlier and independent perceptions of how randomization provides existent minimax solution to a two-person zero-sum game. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Cournot duopoly theory; Black-Scholes formula; Game theory

Scientific theories are like fashions in clothing: sometimes short skirts are in a bull market, sometimes they constitute a good short sale; in one decade the best and the brightest among economists are investing in capital theory, in a different epoch it is linear and dynamic stochastic programming that is all the rage.

Game theory has certainly engaged plenty of our best minds in recent decades. It has produced no dominating equation like the magical Black-Scholes formula of modern finance theory. But game theory virtuosi have been awarded Nobel Prizes by the Olympian Gods of Stockholm.

Johnny von Neumann is for economists the founding father of game theory. Being one of the greatest mathematicians of the last century, von Neumann paid our profession a great compliment when he deigned to honor us with his attention. In fact he paid us at least two other compliments: his model of dynamic balanced growth gave us inequalities–equalities methods of duality which borrowed from his 1928 gem of two-person constant-sum games and speeded up our conquest of linear programming. Furthermore, in a spare moment between quantum physics and computer construction, von Neumann provided an independent discovery of the statistical distribution of the serial correlation coefficient in econometrics. The inventors of the hydrogen bomb, when listed in non-alphabetical order, would read as Stan Ulam, Edward Teller and John von Neumann. Clearly, like Frank Ramsey, von Neumann comes to our inexact science with impeccable credentials.

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PII: S0922-1425(01)00069-X

Before 1928, economics had dabbled in game theory. We called it the theory of duopoly. And way back in 1838, A.A. Cournot had given for it what after 1950 would come to be known as the Nash-equilibrium for a two-person or *n*-person non-cooperative game. My generation learned about this from an appendix in Ed Chamberlin's (1933) classic Theory of Monopolistic Competition. To those in this audience too young to remember the Beatles, here is the canonical Cournot scenario.

Two or more producers of mineral water pool together their respective outputs of q_1 and q_2, and their total gets auctioned off a common price p which obeys the following *linear* demand curve

$$p = 1 - q_1 - q_2 \tag{1}$$

If each has zero costs of production, what will be the best $q_1^*$and q_2^* for each separately? Cournot gives the precise 1950 Nash solution for a non-cooperative game, the roots of

$$p^* = \tfrac{1}{3}, \qquad q_1^* = \tfrac{1}{3} = q_2^* \tag{2a}$$

$$0 = 0 = \partial\frac{(pq_j)}{\partial q_j} = \partial\frac{([1 - q_1 - q_2]q_j)}{\partial q_j}, \quad j = 1, 2 \tag{2b}$$

This Cournot–Nash solution is not Pareto-optimal. Disregarding consumer's interest, Producers' Pareto-optimality necessitates

$$\hat{p}^* = \tfrac{1}{2} > p_* = \tfrac{1}{3}, \qquad \hat{q}_1^* + \hat{q}_2^* = \tfrac{1}{2} \tag{3a}$$

$$\hat{p}^*(\hat{q}_1^* + \hat{q}_2^*) = \tfrac{1}{4} > p^*(q_1^* + q_2^*) = \tfrac{2}{9} \tag{3b}$$

The symmetric case of this would be

$$\hat{q}_1^* = \tfrac{1}{4} = \hat{q}_2^* < \hat{q}_1^* = \tfrac{1}{3} = \hat{q}_2^* \tag{3c}$$

If one seller continues to ignore his own influence on the other's q_j in the Nash manner, then the alert other seller can benefit from the asymmetric von Stackelberg solution

$$0 = \partial\frac{(pq_1)}{\partial q_1} = \partial\frac{[(1 - q_1 - \overline{q}_2)q_2]}{\partial q_2} \tag{4a}$$

for the passive Nash actor and, for his active opponent who suppose that Eq. (4a) does hold in the form of $q_1 = (1 - q_2)/2$

$$0 = \partial\frac{(pq_2)}{\partial q_2} = \partial\frac{[(1 - 1/2 + 1/2q_2 - q_2)q_2]}{\partial q_2} \tag{4b}$$

with a resulting joint solution of

$$\overline{q}_1^* = \tfrac{1}{4}, \qquad \overline{q}_2^* = \tfrac{1}{2}, \qquad \overline{p}^* = \tfrac{1}{4} < p^* = \tfrac{1}{3} < \hat{p}^* = \tfrac{1}{2} \tag{4c}$$

$$\tfrac{2}{9} = p^*(q_1^* + q_2^*) < \overline{p}^*(\overline{q}_1^* + \overline{q}_2^*) = \tfrac{3}{16} < \hat{p}^*(\hat{q}_1^* + \hat{q}_2^*) = \tfrac{4}{16} \tag{4d}$$

von Neumann's 1928 and 1942 two-person constant-sum game differs from the Cournot–Nash non-cooperative game *sans* constant-sum, and therefore the von Neumann–Borel–Fisher neat theorem does *not* apply.