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Profit allocation of independent power producers based on cooperative Game theory

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Abstract

With the development of deregulation, the retail market is being formed. The independent power producers (IPPs) can contact the customers and sell electric power to them directly to obtain the profits because IPPs can provide electricity at cheaper prices to the customers than the utilities can. If IPP can obtain further more profit through collaborating with other ones in some coalition, it will prefer to collaborate to form this coalition rather than participating individually. In coalition, also the problem of how to allocate profit rationally for each IPP should also be solved. In this paper, we discuss the cooperation of IPPs in retail market and give a formulation about the calculation of IPPs profits. After that, based on Game theory, we propose a scheme to decide the profit allocation of each IPP in the coalitions rationally and impartially.

Keywords: Retail market; Independent power producer; Game theory; Coalition; Core; Shapley value; Nucleolus

1. Introduction

Electric utilities have experienced a period of rapid changes especially in market structure and regulatory issues in many parts of the world. Because of the emergence of independent power producers (IPPs) as well as the changing structure of the electricity supply industry, the electric power industry has entered an increasingly competitive environment under which it becomes more realistic to improve economics and reliability of power systems by enlisting market forces [1–4]. For example, to maximize his payoffs, a player (e.g. a utility) seeks to displace expensive generation by importing power from neighboring players with lower cost energy. Likewise, a player (e.g. a IPP) with excess generation capacity can choose to export power and receive an immediate return on its investment [5–8].

Up to now, a considerable number of literature addressing the competition and deregulation issues has been published [9–16], however, the studies on cooperation of IPPs in retail market have not been carried out. In the retail market, IPPs can contact with the customers and sell electric power to them directly to obtain the profits because IPPs can provide cheaper electricity prices to the customers than the utilities can. For the reason of driving for maximum profits as much as possible for the IPPs, the following case can be considered, that is: if IPP can obtain further more profit through collaborating with other ones to form a coalition, it will prefer to collaborate to form this coalition rather than participating individually. Different IPPs will obtain different profits in various coalitions. Though one IPP can obtain the best solution in a coalition, other ones in the same coalition must not obtain their best solutions. How to obtain an equilibrium strategy is considered as a cooperative game problem of coalition form (or normal form) in Game theory [17,18].

In this paper, we discuss the cooperation of IPPs, and propose a method to decide the formation of coalition. Based on Game theory, we get the rational and impartial profit allocation of each IPP in the coalition.

2. Mathematical formulation of problem

In a retail market, the IPPs sell electricity to the customers directly at some sales prices which are made in the contracts. Because electric power is transferred form generation to load by transmission and distribution lines which belong to the utility, the IPPs have to pay the wheeling charges to the utility for the transmission [19,20]. Therefore, the profit of IPP i per hour can be expressed as

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follows

$$v(i) = \rho_i P_i - f_i(P_i) - \omega P_i \tag{1}$$

$$f(i) = a_i P_i^2 + b_i P_i + c_i \tag{2}$$

where

$v(i)$	profit of IPP i (Yen/h)
ρ_i	sales price to customer of generator i (Yen/MW h)
P_i	scheduled effective power of generator i (MW/h)
ω	charge rate of wheeling (Yen/MW h)
$f(i)$	generation cost function of IPP i (Yen/h)
a_i, b_i, c_i	coefficients of generator i

If the IPPs cooperate with others in providing electricity to a customer together, the coalition is formed. In the coalition, to reduce the generation cost, the IPPs will allocate their generation again based on the law of equal incremental fuel cost as follows [21]

$$f_i(P_i) = a_i P_i^2 + b_i P_i + c_i \tag{3}$$

$$\frac{\mathrm{d}f_i}{\mathrm{d}P_i} = 2a_i P_i + b_i = \lambda \tag{4}$$

$$P_i = \frac{\lambda - b_i}{2a_i} \tag{5}$$

$$\begin{aligned} P_1 + P_2 + \cdots + P_n &= \frac{\lambda - b_1}{2a_1} + \frac{\lambda - b_2}{2a_2} + \cdots + \frac{\lambda - b_n}{2a_n} \\ &= \frac{\lambda}{2}\sum_{i=1}^{n}\frac{1}{c_i} - \frac{1}{2}\sum_{i=1}^{n}\frac{b_i}{c_i} = P_R \end{aligned} \tag{6}$$

$$p_i = \frac{1}{2c_i}\frac{\sum_{i=1}^{n}(b_i/c_i) + 2P_R}{\sum_{i=1}^{n}(1/c_i)} - \frac{b_i}{2c_i} \tag{7}$$

where

λ	Lagrange coefficient
P_R	total load (MW)
p_i	effective power of generator i after cooperation (MW/h)

Therefore, in a coalition S whose number of IPPs is m, the profit of all IPPs are expressed by

$$v(S) = \sum_{i=1}^{m}(\rho_i P_i) - \sum_{i=1}^{m} f_i(p_i) - \sum_{i=1}^{m}\omega p_i \tag{8}$$

When considering the power flow in transmission lines, with the load re-allocate in the cooperation between the IPPs, there is possibility that the transmission lines overload. In this case, suppose β is the overload rate (Yen/MW h) that has been given by the transmission service sector, the profit of coalition S is expressed as follows.

$$v(S) = \sum_{i=1}^{m}(\rho_i P_i) - \sum_{i=1}^{m} f_i(p_i) - \sum_{i=1}^{m}\omega p_i - \sum_{i=1}^{m}\beta p_i \tag{9}$$

3. Profit allocation in cooperative game

The IPPs join the market like the players join the game. Based on Game theory, there are two types of games: noncooperative game and cooperative game. Noncooperative games are played by players who choose their strategies independently, whereas in cooperative games coalition formation and joint actions are allowed and generally preferred to acting alone.

Analysis in cooperative Game theory is centered around two major issues: coalition formation and distribution of wealth gained through cooperation. Especially when it comes to the (re)distribution of gains that a group of players (coalition) has achieved, the availability of a linearly transferable commodity (such as money) is crucial. Depending on whether there exists such a commodity which makes it possible that players compensate each other for sacrifices in pursuing a common goal, different methods of analysis are required. Thus we will make a distinction between side-payment (or TU: transferable utility) and non-side-payment (NTU) games. In this paper we will primarily deal with TU-game. In this paper, we consider the cooperation of IPPs as a TU-game.

In a cooperative game, when the IPPs cooperate to form some coalitions, it is sure that different IPPs will obtain different profits in different coalitions. Though one IPP can obtain its maximum profit in some coalition, it does not mean that the other ones can obtain their maximum profits in the same coalition. Therefore, it is important to decide how to obtain an equilibrium strategy for every one. We consider the cooperation between any players in the games is allowed. This means that binding contracts can be made, that correlated mixed strategies are allowed, and that profits can be transferred from one player to the other. In this paper, Game theory is introduced and the above problem is considered as a TU cooperative game.

Analysis in cooperative Game theory is centered on two major issues: coalition formation and distribution of wealth gained through cooperation [17]. If the participants can obtain more profits through collaborating together than before, they will try their best to form a coalition rather than to participate the game individually. Every participant wants to obtain its maximum profit in the coalition, therefore, the satisfactory and reasonable scheme of allocation of profits in the coalition for each one becomes very important.

In this paper, the concepts of the core, Shapley value, kernel and nucleolus are introduced below [17,18,22,23].