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A QBD approach to evolutionary game theory

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Abstract

Game theory is extensively used in economics to predict the best strategies in an evolutionary process of buying/selling, bargaining or in stock market. Many game solvers in the literature use simulation or even experimental games (pay the players). In general simulation takes a huge time and experimental games are very expensive. In this paper, we model the 2×2 non-symmetric game and the 3×3 symmetric game as finite, state dependent quasi-birth-and-death processes. We propose solution procedures based on the block Gaussian elimination for the 2×2 game and the block Gauss–Seidel iteration method for the 3×3 game. Our solver is a powerful tool that gives a probability distribution on the set of strategies available in the game, which helps to identify the best strategies. Furthermore, our game solver is very effective in terms of time and cost. We provide some illustrative examples.

Keywords: Evolutionary game theory; QBD; Hessenberg matrix; Block Gaussian elimination; Block Gauss–Seidel iteration method

1. Introduction

Game theory is concerned with decision situations wherein one party is in conflict with another. Competitive situations arise in almost every facet of human activity—in parlor games, sports, military strategy, and business. In all of these situations, the results achieved depend on both our own action and that of our competitor. There are many features common to both simple games and complicated conflicts in business and industry. For this reason, knowledge of the theory of

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games should be helpful to the decision maker who continually faces competitive situations in business, industry, and government.

One way to describe a game is by listing the players (or individuals) participating in the game, and for each player, listing the alternative choices (called actions or strategies) available to that player. In the case of a two-player game, the actions of the first player form the rows, and the actions of the second player the columns, of a matrix. The entries in the matrix are two numbers representing the payoff to the first and second player, respectively. A very famous game is the Prisoner's Dilemma game. In this game the two players are partners in a crime who have been captured by the police. Each suspect is placed in a separate cell, and offered the opportunity to confess to the crime. The game can be represented by the following matrix of payoffs

	Not confess	Confess
Not confess	(5,5)	(0,10)
Confess	(10,0)	(1,1)

Note that higher numbers are better (more payoff). If neither suspect confesses, they go free, and split the proceeds of their crime which we represent by 5 units of utility for each suspect. However, if one prisoner confesses and the other does not, the prisoner who confesses testifies against the other in exchange for going free and gets the entire 10 units of utility, while the prisoner who did not confess goes to prison and gets nothing. If both prisoners confess, then both are given a reduced term, but both are convicted, which we represent by giving each 1 unit of payoff: better than having the other prisoner confess, but not so good as going free. This game has fascinated game theorists for a variety of reasons. It is a simple representation of a variety of important situations. Consider, for example, two wholesalers competing through their respective supermarket chains. Each fall they must decide on whether they will conduct a promotion campaign the following winter. The larger wholesaler attempts to formulate his decision problem in terms of a two-person game. From past records he knows that in general his chain handles 60% of what he at first considers a fixed segment of the business and his competitor 40%. If he conducts a promotion campaign and his competitor does not, he attracts business not only from his competitor, but also from the other independent stores, and the combined volume of these two major wholesalers is increased by 10 units, with his volume increased 30 units and the competitor's volume decreased 20 units. A similar relationship holds if the competitor is the only one to conduct a campaign. If both wholesalers conduct campaigns they both lose 10 units from their income under routine operations. The wholesaler decides to formulate the problem as a non-zero-sum game. He uses as utility units the volume of business, measured in thousands of dollars, less the cost of a promotion campaign if such is conducted. The result is the following table, where his firm is represented by player 1.

	No promotion	Promotion
No promotion	(60,40)	(40,70)
Promotion	(90,20)	(50,30)

This is a two-person, non-zero-sum game of the prisoner's dilemma type. The "promotion" strategy of each player dominates the other strategy, and so the game has only one equilibrium