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Quantum social game theory

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Abstract

Most game-theoretic studies of strategic interaction assume independent individual strategies as the basic unit of analysis. This paper explores the effects of non-independence on strategic interaction. Two types of non-independence effects are considered. First, the paper considers subjective non-independence at the level of the individual actor by looking at how choice ambivalence shapes the decision-making process. Specifically, how do alternative individual choices superpose with one another to "constructively/destructively" shape each other's role within an actor's decision-making process? This process is termed as quantum superposition of alternative choices. Second, the paper considers how inter-subjective non-independence across actors engenders collective strategies among two or more interacting actors. This is termed as quantum entanglement of strategies. Taking into account both types of non-independence effect makes possible the emergence of a new collective equilibrium, without assuming signaling, prior "contract" agreement or third-party moderation, or even "cheap talk". I apply these ideas to analyze the equilibrium possibilities of a situation wherein N actors play a quantum social game of cooperation. I consider different configurations of large-N quantum entanglement using the approach of density operator. I specifically consider the following configurations: star-shaped, nearest-neighbors, and full entanglement.

1. Introduction

Much of conventional game theory postulates that all observed phenomena of strategic decisions should be explained—are explainable—in terms of the strategic interactions among independent individuals. In this paper I explore how actors' non-independence—more specifically, strategic quantum entanglement—shapes their mutual strategic interactions in situations of social games with a large number of participants. Two types of non-independence effects are considered.

First, the paper considers non-independence at the level of the subjective individual actor by specifically looking at how alternative choices "interfere either constructively or destructively" with one another as part of the process of decision making. This is termed as quantum superposition of strategic choices of an individual actor. Put more simply, a decision maker is not always sure about the analytic delineation of his/her decision choices. An actor might be in a situation where it is not possible to choose unambiguously among the strategic options that are available to him/her in making the decision. He/she can only ascertain that the choice has to

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be some sort of a combination of various pure strategies and that these various choices can positively/negatively shape each other's contribution in the decision-making process. This ambiguity is different from the concept of mixed strategies that play an important role in conventional notions of equilibrium (such as Nash equilibrium) in game theory. A key difference is that in a quantum super-position the decision maker is not randomizing in the sense of mixed strategies. Rather, all pure strategies not only equally contribute to shape the decision-making process, but also either sub-additively or super-additively interfere with each other's contribution to weaken/enhance each other's contribution. This effect of non-independence at the level of individual actor cannot be accounted for in conventional decision-theoretic approaches because it violates three of its key underlying assumptions—namely, that: (1) the decision-maker is able to clearly delineate his/her choice options, (2) he/she can attribute well defined probability estimates (between 0 and 1, inclusive) of these options, and (3) the probabilities of all options in the choice set add up exactly to 1.

Second, the paper considers how strategic inter-subjective non-independence—quantum entanglement—among actors engenders a collective strategy, without assuming signaling, prior "contract" type of arrangement, any form of third-party mediation, or even "cheap talk". This is termed as quantum entanglement of strategies. In quantum entanglement, collective behavior of interacting actors cannot be decomposed (analyzed) into individual contributions of the players without completely deconstructing the strategic game itself. The whole (the game) is more than just the sum of the individual parts (the individual strategies and beliefs of the players) and their mutual strategic interactions. Conceptually speaking, strategic quantum entanglement is based on a sense of "We-ness" mode, which is distinct from the sense of "I-ness" mode prevalent in conventional game theory and rational choice theory. This notion of collective choice is different from the notion of correlated strategies in the sense that the latter are pure, if correlated, individual strategies, whereas in a situation of quantum entanglement the actors lose their strategic individuality.

These ideas and their implications are formally expressed and explored in the new game-theoretic approach termed as Quantum Game Theory (QGT).[1] Briefly described, in addition to providing a formalism through which strategic non-independence can be studied, QGT can account for strategic outcomes that are unpredictable or even forbidden in conventional game theory. One such example is a lifting of equilibrium indeterminacy that inheres in many game-theoretic approaches. For example, in a quantum game of trust, strategic entanglement allows the emergence of trust between two or more actors without a need for signaling, strategic reassurance, prior third-party mediation (such as in those leading to Aumann's notion of "correlated" equilibrium), or even "cheap talk" [11]. Although the players always act in a calculative way in pursuing their respective goals they not only end up avoiding the worst outcomes, but also achieve Pareto-optimality.

In this paper, I use the approach of density operator to study the strategic behavior of a large N ensemble of strategic actors—that is, the quantum statistical properties of a strategic game. The density operator contains the usual probabilistic information about the full set of observable strategies of the system. However, because the density operator (as it will be discussed down below) has both diagonal and off-diagonal elements it not only plays with respect to observable strategies the same role as usual probability distributions do in conventional game-theoretic approach. The density operator also shows the non-classical contributions due to quantum entanglement—this is what makes quantum games truly non-classical. The value-added of this paper consists of two contributions—(1) the consideration of a large-N strategic game (with N actors), and, (2) various types of quantum entanglement configuration. This paper hence bridges quantum statistical mechanics and quantum information to study the conditions of equilibrium in social games of cooperation.

The paper first introduces the two key notions of quantum superposition of an actor's strategic alternatives and quantum entanglement of the choices of interacting players.[2] The following section presents and solves the model of a quantum social game of cooperation based on a prisoner's dilemma stage game. Three cases of entanglement are considered depending on whether all N players are fully quantum-entangled with each other, quantum entanglement occurs between one player and all $(N-1)$ other players while the latter are

[1]For illustrative works, see Refs. [1–10].

[2]See Ref. [11] for related work on quantum game of trust where much of the formalism has already been introduced. This is also similar to the formalism that Marinatto and Weber [12] use.