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Analytic expressions for the BCDMEM model of recognition memory

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Abstract

We introduce a Fourier transformation technique that enables one to derive closed-form expressions of performance measures (e.g., hit and false alarm rates) of simulation-based models of recognition memory. Application of the technique is demonstrated using the bind cue decide model of episodic memory (BCDMEM; [Dennis, S., & Humphreys, M.S. (2001). A context noise model of episodic word recognition. *Psychological Review*, *108*(2), 452–478]). In addition to reducing the time required to test the model, which for models like BCDMEM can be excessive, asymptotic expressions of the measures reveal heretofore unknown properties of the model, such as model predictions being dependent on vector length.

Keywords: Recognition memory; Cognitive modeling; Fourier transformation; Signal detection theory

1. Introduction

A number of simulation-based models of recognition memory have been proposed in recent years. Examples include bind cue decide model of episodic memory (BCDMEM) (Dennis & Humphreys, 2001), REM (Shiffrin & Steyvers, 1997), SAM (Shiffrin, Ratcliff, & Clark, 1990), MINERVA (Hintzman, 1988), and SLiM (McClelland & Chappell, 1998). The popularity of this style of modeling is due in part to the fact that computing technologies permit both flexibility in model design and precision in implementation. A model can be built that incorporates realistic assumptions about the processes underlying encoding, storage, and retrieval. With the aid of random number generators, a model's predictions are obtained as sample averages over a large number of simulation replications.

In this paper, we introduce a technique for deriving closed-form solutions of these models. It involves performing a Fourier transform (FT) on a model, which in essence requires one to rewrite the model in another form. The value of the technique is at least two-fold. Most importantly, the asymptotic expressions provide insights into properties of the model that are difficult to glimpse in the model's original formulation. In addition, the excessive computing time required to derive model predictions and estimate model parameters in simulation-based models is eliminated, thereby facilitating model evaluation. We demonstrate the application of the technique and what can be learned with it using BCDMEM.

BCDMEM is a model of episodic recognition memory that, among other things, simulates human performance in tasks in which participants have to discriminate old from new words after a study phase in which only the old words were presented. The contextual information accompanying each encounter with a word (i.e., episodes) is fundamental to the model and its behavior. Context is encoded with the word and used to discriminate old from new ones during recognition. The essence of the context-based recognition process is briefly reviewed.

Associated with each word is a context vector made of multiple feature units, each taking on the value of either 1 (active, or presence of the feature) or 0 (inactive, or absence of the feature). At study, presentation of a word activates a noisy context vector representing the current experimental context. As a result, some of the feature units in the context

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vector are "learned", meaning that the corresponding feature values become stabilized or permanent and are stored in memory along with the word. Context vectors can undergo spontaneous forgetting, with the value of each feature unit of the vector decaying from 1 to 0. During recognition, when a test word is presented, the memory retrieval process is activated and the context vector corresponding to the test word is retrieved from memory. Simultaneously, the current experimental context vector is reinstated in memory. If the test word is old (i.e., presented during study phase), the overlap between the retrieved context and the reinstated context should be maximal. If the test word is new (i.e., a distracter), then the retrieved context vector would randomly overlap with the reinstated context vector, which in this case would consist of all the contexts in which the test word has been encountered. On both types of trials, the two context vectors, retrieved and reinstated, are compared against each other, feature by feature, to determine whether the test word should be judged old or new.

The model has five parameters: d, p, r, s, v. The parameter d is the contextual reinstatement parameter, and represents the probability that an originally active feature unit in the experimental context vector fails to get reinstated, so becomes inactive in the recognition process. The parameter p is the context noise parameter, and denotes the probability that any feature element of a context vector is active as a result of previous learning. The learning rate parameter, r, stands for the probability that each new feature in the current context vector is copied successfully (i.e., learned) to the memory image. The sparsity parameter, s, represents the probability that each feature in the context vector is turned on spontaneously, independent of prior learning. Finally, the vector length parameter, v, represents the number of distinctive binary features that make up the context vector.

In the model, the recognition decision of 'old' or 'new' is based on the evaluation of a likelihood ratio defined as the probability of observing the data (i.e., test word) under the hypothesis that it is old to the probability of observing the data under the hypothesis that it is new:

$$\text{Decide "old" if } L = \frac{P(data|old)}{P(data|new)} > 1 \text{ and "new" otherwise.} \tag{1}$$

Given that both the retrieved context vector $\boldsymbol{m} = (m_1, \ldots, m_v)$ and the reinstated context vector $\boldsymbol{c} = (c_1, \ldots, c_v)$ are binary (i.e., $m_i = \{0,1\}, c_i = \{0,1\}$), there are four types of match that arise in the comparison between the two vectors. They are denoted by the symbols '00', '01', '10', and '11' where, for example, the '01' match represents the observation of $m_i = 0$ and $c_i = 1$ for the ith feature unit. The likelihood ratio, L, in terms of match types and model parameters is given by (Dennis & Humphreys, 2001, Eq. 6)

$$\begin{aligned} L(\boldsymbol{n} = (n_{00}, n_{01}, n_{10}, n_{11}) | \, \theta = (d, p, r, s), v) \\ = \left[\frac{1 - s + ds(1 - r)}{1 - s + ds}\right]^{n_{00}} \left[\frac{p(1 - s) + ds(r + p - rp)}{p(1 - s) + dsp}\right]^{n_{01}} \\ \times (1 - r)^{n_{10}} \left[\frac{r + p - rp}{p}\right]^{n_{11}}, \end{aligned} \tag{2}$$

where n_{00} denotes the total number of 00 matches, and so on. By definition, the sum of the four match counts must be equal to the vector length parameter v, i.e., $n_{00} + n_{01} + n_{10} + n_{11} = v$. Note that $\boldsymbol{n} = (n_{00}, n_{01}, n_{10}, n_{11})$ is a random variable, and so is the likelihood ratio $L(\boldsymbol{n}| \, \theta, v)$.

2. Statistical formulation of BCDMEM

In this section, we provide a statistical treatment of the context noise process assumed in BCDMEM. This allows us to rewrite the model as a statistical one defined as a parametric family of probability distributions, so as to facilitate application of the FT we carry out next.

2.1. Memory retrieval as a multinomial process

Given the assumption that features are independently sampled during recognition, the matching counts, n_{00}, n_{01}, n_{10}, and n_{11}, follow a multinomial probability distribution because they are discrete random variables. Specifically, for a test item that is old (i.e., presented during study), the probability distribution of the matching counts is given by

$$f(\boldsymbol{n}|\boldsymbol{p}(\theta)) = v! \prod_{\lambda} \frac{p_\lambda(\theta)^{n_\lambda}}{n_\lambda!}. \tag{3}$$

In the equation, $\lambda = \{00, 01, 10, 11\}$ is an index variable representing the four types of pattern matches, and $\boldsymbol{p}(\theta) = (p_{00}(\theta), p_{01}(\theta), p_{10}(\theta), p_{11}(\theta))$ is a vector consisting of four multinomial probability parameters, one for each matching type, defined as: $p_{00}(\theta) = (1 - s)(1 - p) + sd(1 - r)(1 - p)$, $p_{01}(\theta) = (1 - s)p + sd(r + p - rp)$, $p_{10}(\theta) = s(1 - d)(1 - r)(1 - p)$, $p_{11}(\theta) = s(1 - d)(r + p - rp)$. Note that $p_{00}(\theta) + p_{01}(\theta) + p_{10}(\theta) + p_{11}(\theta) = 1$ for all θ, by definition, and that $p_\lambda(\theta = (d, p, r, s))$ does not depend upon the vector length parameter v.

Similarly, for a new item that was not presented during the study phase, the matching counts follow another multinomial probability distribution of sample size v,

$$f(\boldsymbol{n}|\boldsymbol{q}(\theta)) = v! \prod_{\lambda} \frac{q_\lambda(\theta)^{n_\lambda}}{n_\lambda!}. \tag{4}$$

In this equation, the multinomial probability vector $\boldsymbol{q}(\theta) = (q_{00}(\theta), q_{01}(\theta), q_{10}(\theta), q_{11}(\theta))$ is defined as: $q_{00}(\theta) = (1 - s(1 - d))(1 - p)$, $q_{01}(\theta) = (1 - s(1 - d))p$, $q_{10}(\theta) = s(1 - d)(1 - p)$, $q_{11}(\theta) = s(1 - d)p$.

Using the multinomial probabilities $\boldsymbol{p}(\theta)$ and $\boldsymbol{q}(\theta)$, the logarithm of the likelihood ratio, called the *loglikelihood*