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Effect of the depreciation of public goods in spatial public goods games

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ABSTRACT

In this work, the depreciation effect of public goods is considered in the public goods games, which is realized by rescaling the multiplication factor r of each group as $r' = r(\frac{n_c}{G})^{\beta}$ ($\beta \geq 0$). It is assumed that each individual enjoys the full profit r of the public goods if all the players of this group are cooperators. Otherwise, the value of public goods is reduced to r'. It is found that compared with the original version ($\beta = 0$), the emergence of cooperation is remarkably promoted for $\beta > 0$, and there exist intermediate values of β inducing the best cooperation. Particularly, there exists a range of β inducing the highest cooperative level, and this range of β broadens as r increases. It is further presented that the variation of cooperator density with noise has close relations with the values of β and r, and cooperation at an intermediate value of $\beta = 1.0$ is most tolerant to noise.

1. Introduction

Evolutionary game theory [1–7] is widely applied to study the maintenance of cooperation among selfish individuals. The prisoners' dilemma game (PDG) by pairwise interaction [8–10] and public goods game (PGG) by group interaction [11] have been extensively used to investigate altruistic behavior. In a classical public goods game, N players are selected randomly from a large population, and each player has two choices: cooperation and defection. A cooperator will contribute a mount cost to the public pool, while a defector contributes nothing. The accumulative contribution is multiplied by a factor r, then redistributed equally among all the players. In a well-mixed population, it leads to a rock-scissors-paper dynamics when loner strategy is introduced [12]. Considering the limitation of the classical game theory, some mechanisms and theoretical supplement are proposed, such as reputation and punishment [13–23], *network reciprocity* [24–29], voluntary participation [30–33].

Diversity or inhomogeneity has been studied in many works, which mainly focus on the network topology diversity or individual inhomogeneity [34–42]. Santos et al. have introduced social diversity by means of heterogeneous graphs, and concluded that cooperation is promoted by the diversity associated with the number and the size of the public goods game, as well as the individual contribution to each group [27]. Considering the profit diversity of the public goods in reality, group diversity is introduced in the public goods game in which r follows a given distribution among the population [43]. However, in real situations, the value of public goods is not invariable, but evolves due to the external conditions or the reasons of themselves [44,45]. In this work, we study the evolution of cooperation in the spatial public goods games by considering the depreciated effect of public goods. It is assumed that each individual enjoys the full advantage of public goods if all the people are cooperators in a single group, otherwise, the value of the multiplication factor of this group is reduced as a function of

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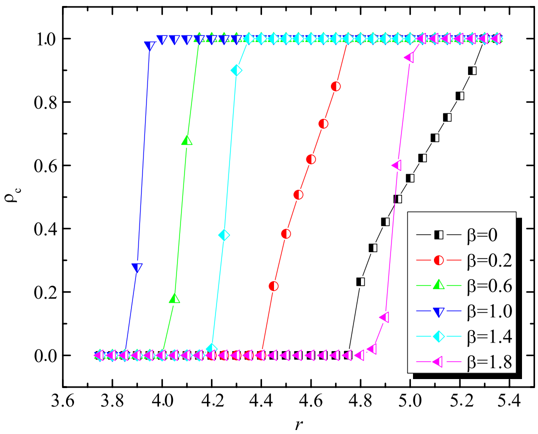

Fig. 1. (Color online) Cooperator density ρ_c as a function of multiplication factor r for different values of β.

$r(\frac{nc}{G})^{\beta}$. It is proved that there exist intermediate values of β inducing the best cooperation, and the range of optimal values of β broadens as r increases. We also investigated the effect of noise on cooperation by adopting this mechanism, and found that cooperation stays at the highest level for most of the range of noise at a moderate value of $\beta = 1.0$.

The paper is organized as follows. In Section 2, a description of the model is proposed in detail. Numerical simulations and the corresponding analysis are presented in Section 3. Conclusions are drawn in Section 4.

2. Models

The public goods game is studied on a square lattice with periotic boundary conditions. Each player can either cooperate or defect, and interacts with its nearest four neighbors. Here each individual collects its payoff only from the group centered on him. Considering the depreciated effect of the public goods, the multiplication factor r_x of the group centered on individual x is rescaled as

$$r'_x = r\left(\frac{n_{cx}}{G}\right)^{\beta} \tag{1}$$

where r is the multiplication factor indicating the full profit of the public goods of each group. n_{cx} denotes the number of cooperators in this group, and G (=5) is the group size. $\beta \geq 0$ is a tunable parameter, and when $\beta = 0$, the system returns to the original version in which r is invariable and has the same value in each group. The payoff of player x is given by

$$P_x = r'_x\frac{n_{cx}}{G} - s_x, \tag{2}$$

where s_x indicates the strategy of x, $s_x = 1$ for a cooperator, and 0 for a defector.

After each time step, a player x will update its strategy by choosing a neighbor y randomly from the neighborhood. We adopt the updating rule depending on their total payoff difference,

$$W(s_x \to s_y) = \frac{1}{1 + \exp[(P_x - P_y)/\kappa]}, \tag{3}$$

where κ denotes the amplitude of noise level. Here we set $\kappa = 0.1$.

3. Simulation and analysis

Simulations are carried out for a population of $N = 200 \times 200$ individuals with the synchronous updating rule. Initially, the two strategies of cooperation (C) and defection (D) are randomly distributed with the equal probability of 1/2. The key quantity for characterizing the cooperative behavior is the density of cooperators ρ_c, which is defined as the fraction of cooperators in the whole population. In the all simulations, ρ_c is obtained by averaging over the last 5000 Monte Carlo (MC) time steps of the total 45 000. Each data point results from an average of 50 realizations.

Fig. 1 shows the variation of ρ_c with r for different values of β. One can see that ρ_c increases with r for each β, and the emergence of cooperation is remarkably promoted compared with the original version ($\beta = 0$). Moreover, $\beta = 1.0$ induces the best promotion, which suggests that there exist moderate values of β for the evolution of cooperation.

Then we study the effect of β on the evolution of cooperation in Fig. 2 for different values of r. It is shown that for each value of r, there exist intermediate values of β leading to the highest cooperative level. And meanwhile, the range of β inducing the best cooperation effect broadens as r increases. Moreover, we can also conclude that the distribution of r has