A cooperative game theory analysis for transmission loss allocation

Delberis A. Lima [a], Javier Contreras [b,*], Antonio Padilha-Feltrin [a]

[a] *Universidade Estadual Paulista, Campus de Ilha Solteira, Av. Brasil 56, CEP 15385-000 Ilha Solteira, SP, Brazil*
[b] *Universidad de Castilla-La Mancha, E.T.S. de Ingenieros Industriales, Campus Universitario s/n, 13071 Ciudad Real, Spain*

Abstract

This paper presents an analysis and discussion, based on cooperative game theory, for the allocation of the cost of losses to generators and demands in transmission systems. We construct a cooperative game theory model in which the players are represented by equivalent bilateral exchanges and we search for a unique loss allocation solution, the Core. Other solution concepts, such as the Shapley Value, the Bilateral Shapley Value and the Kernel are also explored. Our main objective is to illustrate why is not possible to find an optimal solution for allocating the cost of losses to the users of a network. Results and relevant conclusions are presented for a 4-bus system and a 14-bus system.
© 2007 Elsevier B.V. All rights reserved.

Keywords: Loss allocation; Equivalent bilateral exchanges; Core; Shapley Value; Bilateral Shapley Value; Kernel

1. Introduction

Different proposals for the allocation of the cost of losses in electric networks have appeared in the last years for transmission and distribution systems. Clearly, the focus has been mainly on transmission systems, due to the large amount of losses involved and the increasing number of agents with open access to transmission networks. In this way, several proposals have appeared in the technical literature and some comparative studies were presented [4,24]. For distribution systems some proposals have been made, mainly considering the presence of distributed generation [8,26].

Loss allocation methods are aimed at assigning the responsibility of paying for part of the transmission system total losses to each generator and each load. Therefore, several characteristics are important for a suitable method [3], namely: to reflect the magnitude of the power injection at each bus; to consider the relative position of a bus in the transmission network; to be consistent with the solved power flow; to be stable with respect to small alterations of network parameters; to provide effective incentives or disincentives for generators and loads regarding their relative locations in the network; and, finally, it is desirable that the method is simple to understand and implement.

Several methods have been proposed in the last few years, among them: methods based on network matrices [3], methods based on proportional sharing [1,2], methods based on incremental transmission losses [13], methods based on the pro-rata technique [20] and methods based on bilateral contracts [10,12]. A comparative revision of some of these methods is presented in Refs. [11,28] and an analysis of the results is shown in Refs. [23,24]. This paper presents an analysis of the allocation of the cost of losses from the game theory point of view. A related work, presented in Ref. [10], shows an analysis of loss allocation methods in markets with bilateral transactions comparing two game theory-based methods: the nucleolus [29] and the Shapley Value [30]. The two methods in Ref. [10] assume that losses are directly related to bilateral transactions, considering the transactions as players that can form coalitions. This paper has in common with Ref. [10] the use of the Shapley Value to obtain the final loss allocation but, in contrast, uses other concepts, such as the equivalent bilateral exchanges (EBE) [14] considered as players, the Core

* Corresponding author. Tel.: +34 926 295464; fax: +34 926 295361.
E-mail addresses: delberis@aluno.feis.unesp.br (D.A. Lima),
Javier.Contreras@uclm.es (J. Contreras), padilha@dee.feis.unesp.br
(A. Padilha-Feltrin).

[18], the Bilateral Shapley Value [5,6] and the Kernel [7,9,22]. In addition, our paper deals with a pool-based electricity market and reference [10] studies a bilateral electricity market. These results illustrate why there are so many methods for loss allocation and why it is so difficult to get a method that satisfies all the network users. We use a cooperative game theory approach as a well founded economic framework to qualitatively study loss allocation. We have restricted ourselves to apply it to the EBE methodology, since we consider that it presents several advantages with respect to other methods, mainly the non-arbitrary selection of the slack bus. Other currently used loss allocation methods could be analyzed using the same framework.

The loss allocation method proposed is divided into two steps.

Assume a dc model, and find unbundled power flows through the network based on the EBE methodology [14].

Obtain the characteristic function of losses corresponding to an individual EBE or a set of EBEs. In our paper, in order to calculate the characteristic function of losses of an EBE or a set of EBEs, we combine the EBE methodology with the loss formula from a dc load flow model.

The contributions of this paper are two-fold:

(1) To provide a cooperative game theory framework where different existing loss allocation methods can be tested and qualitatively compared.
(2) To obtain game-theoretical economic-based solutions to the loss allocation problem, namely Core, Shapley Value, Bilateral Shapley Value and Kernel.

The paper is organized as follows. Section 2 presents how to unbundle network flows based on the concept of EBE, Section 3 describes the cost allocation game based on EBE and cooperative game theory, Section 4 presents two case studies where several game theory allocation methods are applied, and Section 5 enumerates relevant conclusions. An Appendix A containing relevant definitions and derivations on shift distribution factors and cooperative game theory is included.

2. Allocating network flows

Using EBEs allows determining the power flow distribution for different scenarios of generation and demand.

Define the value of an EBE l [14], GD_l, between the generation at bus i and the demand at bus j by

$$GD_l = \frac{P_{Gi} P_{Dj}}{P_D^{sys}}, \tag{1}$$

where in a lossless network, the system demand is given by $P_D^{sys} = \sum_{j \in \Omega_D} P_{Dj} = \sum_{i \in \Omega_G} P_{Gi}$, and Ω_G and Ω_D are the sets of all generators and demands, respectively. The value GD_l can be viewed as the fraction of the generation P_{Gi} that supplies the demand P_{Dj}, or equivalently, the fraction of the demand P_{Dj} supplied by the generation P_{Gi}. The EBE methodology is a reasonable way to determine the contribution of each generator to each demand. The method represents an advantage in relation to other methods, like ITL, for instance, because it is independent

of the slack bus. In the EBE framework, a slack bus is created for each EBE, which makes the method independent of the selection of a unique slack bus, as shown in Refs. [14,15,17].

Under a dc load flow model, let γ_{lk} denote the generation shift distribution factor (DF) due to EBE l in line k [25,27]. This parameter describes the sensitivity of the power flow through line k, F_k, with respect to the unitary power injection at bus i, under the assumption that the slack bus is located at bus j. See Appendix A.

Using DFs, any power flow in line k can be expressed as a linear combination of shift distribution factors:

$$F_k = \sum_{l \in \Omega_E} \gamma_{lk}, \tag{2}$$

where F_k represents the power flow through line k due to the unitary injection at the bus i and the unitary extraction at the bus j, and Ω_E is the set of all EBEs in the network. Note that F_k is the sum of the flows through line k (sum of all F_{lk} in the Appendix A) due to all EBEs.

Now, similarly to Eq. (2), we can calculate the power flow through line k as a result of all EBEs:

$$P_{F_k} = \sum_{l \in \Omega_E} \gamma_{lk} GD_l, \tag{3}$$

where P_{F_k} is the power flow through line k due to all EBEs in the network. Note that P_{F_k} is the kth element of the vector P_F of line flows described in the Appendix A. Finally, to calculate the effect of a single EBE in the power flow through line k, we obtain:

$$P_{F_{lk}} = \gamma_{lk} GD_l, \tag{4}$$

where $P_{F_{lk}}$ is the contribution of the EBE l to the power flow through line k due to an injection GD_l at bus i and the extraction of the same value at bus j. The buses where there is both generation and demand are considered as buses with a net generation equal to the generation minus the demand in the node. For EBEs purposes, these buses have a net generation that is susceptible to bilaterally exchange power with all the demands of the network.

3. The loss allocation game

We can treat the loss allocation problem as a cooperative game, where the players are represented by the EBEs that have been described in the previous section. The objective of the game is to obtain the total losses of the network and to allocate them using different game theory solution concepts. To do that, we need first to define the value of the function that represents the amount of losses related to an EBE: the characteristic function of losses (see Appendix A). In our game, each EBE is represented by a combination of generation–demand that is associated to a characteristic function of losses. Next, we construct all possible combinations of EBE and the different characteristic functions of losses that they produce: this is called a coalition in cooperative game theory, whose properties are described in detail in the Appendix A.