Computers and Mathematics with Applications 62 (2011) 4207–4221

Contents lists available at SciVerse ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Technical importance ratings in fuzzy QFD by integrating fuzzy normalization and fuzzy weighted average ☆

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ARTICLE INFO

Article history:
Received 28 April 2010
Received in revised form 1 June 2011
Accepted 4 October 2011

Keywords:
Fuzzy QFD
Fuzzy normalization
Fuzzy weighted average
Technical importance ratings
Alpha level sets

ABSTRACT

Fuzzy quality function deployment (QFD) has been extensively used for translating customer requirements (CRs) into product design requirements (DRs) in fuzzy environments. Existing approaches, however, for rating technical importance of DRs in fuzzy environments are found problematic, either incorrect or inappropriate. This paper investigates how the technical importance of DRs can be correctly rated in fuzzy environments. A pair of nonlinear programming models and two equivalent pairs of linear programming models are developed, respectively, to rate the technical importance of DRs. The developed models are examined and illustrated with two numerical examples.

1. Introduction

Quality function deployment (QFD) [1] is a methodology for translating customer requirements (CRs), i.e. the voice of the customer (VoC), into product design requirements (DRs). In this translating process, a large number of subjective judgments have to be made by both customers and QFD team members. Due to uncertainty and vagueness involved in subjective judgments, fuzzy logic has been widely suggested for better capturing the relative importance of CRs and the relationships between CRs and DRs as well as the correlations among DRs. As a result, fuzzy QFD has been developed, researched and extensively applied [2–11].

For fuzzy QFD, one of the key issues is to derive the technical importance ratings of DRs in fuzzy environments and prioritize them so that limited resources such as budget can be reasonably or optimally allocated within DRs in terms of their priorities. Existing approaches for rating the technical importance of DRs in fuzzy environments are found problematic, either incorrect or inappropriate. Therefore, there is a need to develop a correct methodology for rating the technical importance of DRs. This paper investigates how the technical importance of DRs can be correctly rated in fuzzy environments. A pair of nonlinear programming (NLP) models is developed to correctly rate the technical importance of DRs in fuzzy environments, which is then broken down into two equivalent pairs of linear programming (LP) models for solution.

The paper is organized as follows. Section 2 gives a brief introduction to fuzzy sets and fuzzy weighted average that are or will be used in fuzzy QFD. Section 3 presents a literature review on the formulas and approaches for rating the technical importance of DRs in fuzzy environments and points out their incorrectness or inappropriateness. Section 4 develops correct NLP models for rating the technical importance of DRs. Section 5 shows how the NLP models can be simplified as two

☆ The work described in this paper is supported by the National Natural Science Foundation of China (NSFC) under the Grant No.70925004 and also substantially supported by a grant from CityU (project no. 7002571).

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doi:10.1016/j.camwa.2011.10.005

equivalent pairs of LP models for solution. The developed models, linear and nonlinear, are examined and illustrated with two numerical examples in Section 6. The paper concludes in Section 7.

2. Fuzzy sets and fuzzy weighted average

Fuzzy sets were introduced by Zadeh [12]. A fuzzy set is a collection of elements in a universe of discourse, with each element being assigned a value within [0,1] by a specified membership function. It can also be represented using α-level sets. The α-level sets, A_α, of a fuzzy set $\tilde{A}$ are defined as [13]

$$A_\alpha = \left\{x \in X | \mu_{\tilde{A}}(x) \geq \alpha\right\} = \left[\min\{x \in X | \mu_{\tilde{A}}(x) \geq \alpha\}, \max\{x \in X | \mu_{\tilde{A}}(x) \geq \alpha\}\right], \tag{1}$$

where $\mu_{\tilde{A}}(x)$ is the membership function of $\tilde{A}$ and X is the universe of discourse. Accordingly, the fuzzy set $\tilde{A}$ can be equivalently expressed as

$$\tilde{A} = \cup_\alpha \alpha \cdot A_\alpha = \cup_\alpha \alpha \cdot \left[(A)_\alpha^L, (A)_\alpha^U\right], \quad 0 < \alpha \leq 1. \tag{2}$$

Fuzzy numbers are special cases of fuzzy sets, characterized by given intervals of real numbers. The most commonly used fuzzy numbers are triangular and trapezoidal fuzzy numbers, whose membership functions are, respectively, defined as

$$\mu_{\tilde{A}_1}(x) = \begin{cases} (x-a)/(b-a), & a \leq x \leq b, \\ (c-x)/(c-b), & b \leq x \leq c, \\ 0, & \text{otherwise}, \end{cases} \tag{3}$$

$$\mu_{\tilde{A}_2}(x) = \begin{cases} (x-a)/(b-a), & a \leq x \leq b, \\ 1, & b \leq x \leq c, \\ (d-x)/(d-c), & c \leq x \leq d, \\ 0, & \text{otherwise}. \end{cases} \tag{4}$$

For brevity, triangular and trapezoidal fuzzy numbers are often denoted as (a, b, c) and (a, b, c, d). Triangular fuzzy numbers can also be expressed as a special case of trapezoidal fuzzy numbers.

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ be two positive trapezoidal fuzzy numbers. Operations on the two fuzzy numbers, which are often called fuzzy arithmetics, are defined as [13]

Fuzzy addition: $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$;

Fuzzy subtraction: $\tilde{A} - \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$;

Fuzzy multiplication: $\tilde{A} \otimes \tilde{B} \approx (a_1b_1, a_2b_2, a_3b_3, a_4b_4)$;

Fuzzy division: $\tilde{A} \div \tilde{B} \approx \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\right)$.

Fuzzy sets are not easy to compare and often defuzzified for ranking purpose. Defuzzification is a transformation process which converts a fuzzy set into a crisp value. The most commonly used method for defuzzification is the centroid method, which is defined as [14]

$$\bar{x}_0(\tilde{A}) = \frac{\int_{-\infty}^{+\infty} x\mu_{\tilde{A}}(x)dx}{\int_{-\infty}^{+\infty} \mu_{\tilde{A}}(x)dx}, \tag{5}$$

where $\bar{x}_0(\tilde{A})$ is the centroid. In the case that fuzzy sets are characterized by α-level sets without knowing their explicit membership functions, their centroids can be computed by the following expressions [15]:

$$\begin{aligned}\int_{-\infty}^{+\infty} x\mu_{\tilde{A}}(x)dx = {} & \frac{1}{6n}\left[\left((x)_{\alpha_0}^{2U} - (x)_{\alpha_0}^{2L}\right) + \left((x)_{\alpha_n}^{2U} - (x)_{\alpha_n}^{2L}\right) + 2\sum_{i=1}^{n-1}\left((x)_{\alpha_i}^{2U} - (x)_{\alpha_i}^{2L}\right)\right] \\ & + \frac{1}{6n}\sum_{i=0}^{n-1}\left((x)_{\alpha_i}^{U} \cdot (x)_{\alpha_{i+1}}^{U} - (x)_{\alpha_i}^{L} \cdot (x)_{\alpha_{i+1}}^{L}\right),\end{aligned} \tag{6}$$

$$\int_{-\infty}^{+\infty} \mu_{\tilde{A}}(x)dx = \frac{1}{2n}\left[\left((x)_{\alpha_0}^{U} - (x)_{\alpha_0}^{L}\right) + \left((x)_{\alpha_n}^{U} - (x)_{\alpha_n}^{L}\right) + 2\sum_{i=1}^{n-1}\left((x)_{\alpha_i}^{U} - (x)_{\alpha_i}^{L}\right)\right], \tag{7}$$

where $\alpha_i = \frac{i}{n}$, $i = 0, \ldots, n$. In the case of $(x)_{\alpha_n}^L = (x)_{\alpha_n}^U$, the centroid can be computed by

$$\bar{x}_0(\tilde{A}) = \frac{1}{3} \cdot \frac{\left((x)_{\alpha_0}^{2U} - (x)_{\alpha_0}^{2L}\right) + 2\sum_{i=1}^{n-1}\left((x)_{\alpha_i}^{2U} - (x)_{\alpha_i}^{2L}\right) + \sum_{i=0}^{n-1}\left((x)_{\alpha_i}^{U} \cdot (x)_{\alpha_{i+1}}^{U} - (x)_{\alpha_i}^{L} \cdot (x)_{\alpha_{i+1}}^{L}\right)}{\left((x)_{\alpha_0}^{U} - (x)_{\alpha_0}^{L}\right) + 2\sum_{i=1}^{n-1}\left((x)_{\alpha_i}^{U} - (x)_{\alpha_i}^{L}\right)}. \tag{8}$$