Electric Power Systems Research 80 (2010) 1082–1088

Contents lists available at ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

On impedance matching and maximum power transfer

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ARTICLE INFO

Article history:
Received 19 October 2009
Received in revised form 27 January 2010
Accepted 27 January 2010
Available online 24 February 2010

Keywords:
Power systems
Maximum power transfer
Impedance matching
Network equivalent
Predictor–corrector framework

ABSTRACT

This paper investigates the relationship between the impedance matching and the maximum power transfer problems, and presents a predictor–corrector framework for fast estimation of the maximum power transfer limit for the load increase pattern with a common scaling factor. First, a network equivalent technique is presented and a "decoupled" equivalent network is obtained for every load bus. Second, a special form of impedance matching condition is derived for the maximum power transfer problem with an unlimited load variation pattern. Though this is an unrealistic case in power systems, it might have a profound physical mechanism and lead to an interesting explanation and application for the presented equivalent technique. Third, this paper discusses the relationship between the impedance matching and the maximum power transfer problem for the load increase pattern with a common scaling factor. Finally, a predictor–corrector framework is introduced for fast estimation of the maximum power transfer limit for the load increase pattern with a common scaling factor.

1. Introduction

Due to economic, technical and environmental concerns, today's power systems are being forced to operate closer to their loadability limits. As a result, the power transfer capability of an interconnected power system has been becoming an important concern of both system planners and operators. Therefore, how to determine the maximum power transfer limit of an interconnected power system has become an increasingly important issue in power system planning and operation.

In modern power systems, the increasing penetration of renewable energy sources is putting power system security in a challenge because of large uncertainties. Excessive power injections of renewable sources could cause an undue risk of system overloads, voltage collapse, or even blackouts due to deficient power transfer capabilities. Therefore, accurate evaluation of transfer capability is essential to maximize utilization of existing transmission grids while maintaining system security.

In general, the power transfer of an interconnected power system may be limited by the physical and electrical characteristics of the systems including any one or more of thermal, voltage and stability limits [1]. When the dynamic or transient stability constraints are considered, it will be very difficult to determine the maximum power transfer limit for online applications [2–4]. Therefore, in the last decades, considerable attentions were mainly paid to the calculation of the maximum power transfer limit with static security constraints [5–8], and the conventional *P*–*V* and *Q*–*V* curves are widely used as a tool for off-line application in utility industries [9,10].

In recent years, due to the fast development of synchronized phasor measurement technologies, the measurement-based methods have received considerable attentions for online estimation of the maximum power transfer limit [11–18]. In essence, this type of method is based on the well-known impedance matching principle. The measured data are used to obtain the Thevenin equivalent of the system, as seen from the load bus under study, and the apparent impedance of the load. When the Thevenin impedance matches the load impedance in magnitude, the maximum power transfer is reached.

The purpose of this paper is to discuss the relationship between impedance matching and maximum power transfer, and thus propose a predictor–corrector framework for fast estimation of the maximum power transfer limit for the load increase pattern with a common scaling factor. First, a network equivalent technique is presented. In Section 2, a special form of impedance matching condition is derived for the maximum power transfer problem with the unlimited load variation pattern. Section 4 discusses the maximum power transfer problem for the limited load increase pattern. In Section 5, a predictor–corrector framework is introduced for fast estimation of the maximum power transfer limit for the load

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0378-7796/$ – see front matter
doi:10.1016/j.epsr.2010.01.015

increase pattern with a common scaling factor. Finally, concluding remarks are given.

2. Network reduction and equivalent

The buses in an interconnected power system can generally be classified into three categories: generator bus, load bus and tie bus (without generators and loads connecting to it). A generator bus becomes a load bus if the connected generator reaches its capacity limit and loses its voltage regulation capability. Because the injection currents to the tie buses are zero, the injection currents into the three types of buses can be generally expressed [19]:

$$\begin{bmatrix} -I_L \\ 0 \\ I_G \end{bmatrix} = \begin{bmatrix} Y_{LL} & Y_{LT} & Y_{LG} \\ Y_{TL} & Y_{TT} & Y_{TG} \\ Y_{GL} & Y_{GT} & Y_{GG} \end{bmatrix} \begin{bmatrix} V_L \\ V_T \\ V_G \end{bmatrix} \tag{1}$$

where the Y matrix is known as the system admittance matrix, V and I represent the voltage and current vectors, and the subscript L, T, and G stand for load bus, tie bus, and generator bus, respectively.

It can be seen from (1) that, the load bus voltages can be solved by:

$$V_L = H_{LG}V_G - Z_{LL}I_L \tag{2}$$

where

$$Z_{LL} = (Y_{LL} - Y_{LT}Y_{TT}^{-1}Y_{TL})^{-1} \tag{3}$$

$$H_{LG} = Z_{LL}(Y_{LT}Y_{TT}^{-1}Y_{TG} - Y_{LG}) \tag{4}$$

Clearly, the first term on the right side of (2) is the open-circuit voltage vector for load buses:

$$E_{open} = H_{LG}V_G \tag{5}$$

Thus, the equivalent equation

$$V_L = E_{open} - Z_{LL}I_L \tag{6}$$

can be obtained for all load buses, and Fig. 1 shows the corresponding equivalent circuit.

Let $Z_{LL,i}$ represents the ith row of Z_{LL}. Obviously, Eq. (6) can be rewritten as follows:

$$V_{Li} = E_{openi} - Z_{eqi}I_{Li} \quad (i = 1, 2, \cdots, n) \tag{7}$$

where Z_{eqi} represents the equivalent impedance seen from the load i:

$$Z_{eqi} = \frac{Z_{LL,i}I_L}{I_{Li}} \quad (i = 1, 2, \cdots, n) \tag{8}$$

Consequently, as shown in Fig. 2, a "decoupled" equivalent circuit can be obtained.

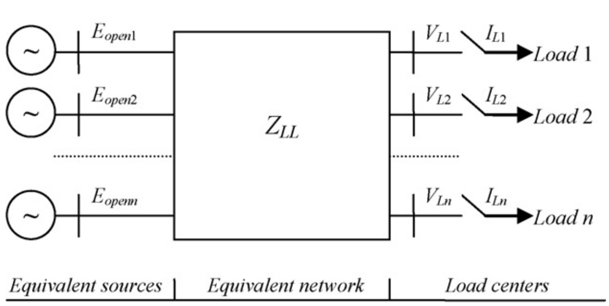

Fig. 1. The equivalent circuit for all load buses.

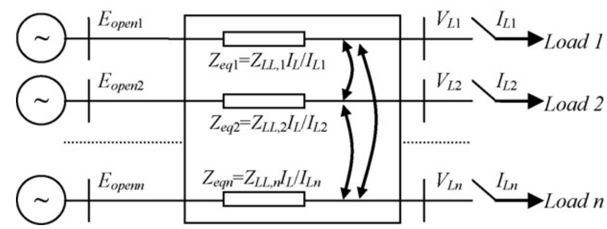

Fig. 2. The "decoupled" equivalent circuit.

Let Z_{LLii} and Z_{LLij} represent the ith diagonal element and the i–j element of Z_{LL}, respectively. Eq. (8) can be rewritten as follows:

$$Z_{eqi} = Z_{LLii} + \sum_{j=1, j \neq i}^{n} Z_{LLij}\frac{I_{Lj}}{I_{Li}} \quad (i = 1, 2, \cdots, n) \tag{9}$$

It can be seen from (9) that the equivalent impedance consists of two terms: one is the self-impedance Z_{LLii}, and the other is related to both the mutual-impedance Z_{LLij} and the load currents.

3. Maximum power transfer for the unlimited load variation pattern

3.1. Theoretical analysis

For the equivalent circuit shown in Fig. 1, assume that the load power can be changed freely; the maximum power transfer problem can be expressed as follows:

$$\begin{aligned} \underset{I_L^r, I_L^x}{Max}\ f(I_L^r, I_L^x) &= real(I_L^+ V_L) \\ &= real(I_L^+(E_{open} - Z_{LL}I_L)) \\ &= (I_L^r)^T E_{open}^r + (I_L^x)^T E_{open}^x - (I_L^r)^T Z_{LL}^r I_L^r - (I_L^x)^T Z_{LL}^r I_L^x \end{aligned} \tag{10}$$

where the superscript T represents the transpose, (+) denotes the conjugate transpose, Z_{LL}^r is the real part of Z_{LL}, I_L^r and I_L^x are the real and imaginary parts of I_L, and E_{open}^r and E_{open}^x are the real and imaginary parts of E_{open}.

In order to find the maximum active load power, the following equations must be satisfied:

$$\frac{df(I_L^r, I_L^x)}{dI_L^r} = E_{open}^r - Z_{LL}^r I_L^r - (Z_{LL}^r)^T I_L^r = 0 \tag{11}$$

$$\frac{df(I_L^r, I_L^x)}{dI_L^x} = E_{open}^x - Z_{LL}^r I_L^x - (Z_{LL}^r)^T I_L^x = 0 \tag{12}$$

Thus, we can get the load current $I_{L\max}^r$ and $I_{L\max}^x$ for the maximum power transfer level:

$$I_{L\max}^r = (Z_{LL}^r + (Z_{LL}^r)^T)^{-1} E_{open}^r \tag{13}$$

$$I_{L\max}^x = (Z_{LL}^r + (Z_{LL}^r)^T)^{-1} E_{open}^x \tag{14}$$

Once both $I_{L\max}^r$ and $I_{L\max}^x$ are obtained, the maximum transfer power can be calculated easily using (10).

Combining (13) and (14), we can get:

$$I_{L\max} = (Z_{LL}^r + (Z_{LL}^r)^T)^{-1} E_{open} \tag{15}$$

According to (6), the following equation can be obtained:

$$Z_{L\max}I_{L\max} = V_{L\max} = E_{open} - Z_{LL}I_{L\max} \tag{16}$$

where Z_{Lmax} is the load impedance matrix for the maximum power transfer level.