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Income inequality, quasi-concavity, and gradual population shifts☆

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Abstract

An income distribution is a *mixture* of two given income distributions if the relative frequency it associates with each income level is a convex combination of the relative frequencies associated with it by the given two income distributions—e.g., the income distribution of a country is obtained as a mixture of the income distributions of its regions. In this article, it is established that all inequality measures commonly considered in the literature—the class of decomposable inequality measures and the class of normative inequality measures based on a social welfare function of the rank-dependent expected utility form—satisfy quasi-concavity properties, which imply, loosely speaking, that mixing income distributions increases inequality. These quasi-concavity properties are then shown to greatly reduce the possible patterns describing the evolution of inequality in the overall income distribution (a mixture) during a process in which population gradually shifts from one of its constituent income distributions to another over time.

Keywords: Inequality; Mixture; Quasi-concavity; Rank-dependent expected utility

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1. Introduction

In the analysis of income inequality, it is often useful to view the income distribution of interest as being composed of several constituent income distributions, e.g., the income distributions

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corresponding to different regions, sectors, or genders. The question of how inequality in the overall income distribution is affected if the constituent income distributions change, has received considerable attention in the form of decomposability analysis.[1] By contrast, the complementary question of how overall inequality changes if the population shares corresponding to the constituent income distributions change, has not been studied much. Nevertheless, the latter question is interesting both from the empirical and the theoretical perspective.

There are several empirical phenomena that involve a shift of the population from one constituent income distribution to another. Take as an example the phenomenon of demographic ageing. In this case, the overall income distribution changes over time because population gradually shifts from the income distribution of working consumers to the income distribution of retired consumers. Another (particularly natural) example is that of a country with two regions that have different population growth rates: here population shifts from the income distribution corresponding to the region with the lower growth rate to that corresponding to the region with the higher growth rate. As a final example, consider the development process studied by Kuznets (1955) which involves a gradual population shift from the income distribution of the agricultural sector to that of the industrial sector.

Besides being of empirical interest, the gradual population shift process is relevant theoretically. In order to see this, assume that the overall income distribution is constituted of two perfectly equal income distributions: one in which everyone has income 10 and another in which everyone has income 50. Now, suppose that we start off with the entire population in the former income distribution, and that population gradually shifts to the latter over time. The income distribution will take, among others, the following three forms at various stages of this simple process:

$$A = \begin{cases} 90\% \text{ has } 10 \\ 10\% \text{ has } 50 \end{cases}, \qquad B = \begin{cases} 50\% \text{ has } 10 \\ 50\% \text{ has } 50 \end{cases}, \qquad C = \begin{cases} 10\% \text{ has } 10 \\ 90\% \text{ has } 50 \end{cases}.$$

Thinking about how inequality evolves as the income distribution changes from A to B and from B to C obviously means thinking about how inequality judgements are influenced by the relative population sizes of the "rich" and "poor." For this reason, this simple case of the gradual population shift process has been considered of importance for the theoretical question of how inequality comparisons ought to be made in the first place. It has been studied in this way by Fields (1987, 1993), among others.

The key to tackling the question of how inequality evolves during a gradual population shift lies in the behaviour of inequality measures with respect to mixing income distributions. Let us first explain what we mean by mixing income distributions. Assuming that income distributions are defined in terms of relative frequencies, each income distribution can be defined as a mixture, i.e., a convex combination, of its constituent income distributions. As an illustration, consider a country with two regions: "region P" and "region Q," representing population shares of α and $1-\alpha$, respectively. Indeed, if p_x and q_x are the proportions of the population with income x in regions P and Q, respectively, then the proportion of the population with income x in the country is equal to $\alpha p_x+(1-\alpha)q_x$. Now, during a gradual population shift process, the income distribution at each stage is a mixture of the income distribution at any earlier stage and the income distribution at any later stage—as an illustration, note that income distribution B in the example above of the simple case of the process, is a fifty-fifty mixture of income distributions A and C. In order to describe the evolution of inequality during a gradual population shift process, the important question is whether income inequality in a mixture is greater than, smaller than, or equal to, income inequality in each

[1] See, e.g., the overview of the literature on inequality measurement by Cowell (2000).