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Notions of equivalence through ratios: Students with and without learning disabilities

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ABSTRACT

Students with learning disabilities (LD) specific to mathematics historically underperform in foundational content such as rational number equivalence. This study examined the strategy usage and multiplicative thinking of three third grade children (i.e., Bill, a child identified as having a learning disability specific to mathematics, Carl, a child labeled as low achieving in mathematics, and Albert, a child labeled as typically achieving) before, during, and after participating in tutoring sessions consisting of student-centered pedagogy and equivalence tasks presented through an underutilized interpretation of rational number: namely, the ratio interpretation. Constant comparison analysis of the children's work during the tutoring sessions as well as responses to tasks during two clinical interviews seemed to indicate that all three children increased their use of viable strategies, with notable differences in the sophistication of the strategies as well as the level of multiplicative thinking utilized before and after the ratio-based tutoring sessions. Yet, Bill's continued use of rudimentary strategies reflects a need for continued research to investigate why the use of such strategies persists and how supporting the development of more sophisticated strategies (especially among children with LD) can be achieved.

Children with learning disabilities (LD) specific to mathematics historically underperform in foundational content such as fraction equivalence (Cawley & Miller, 1989; Grobecker, 2000; Hecht, Vagi, & Torgesen, 2006; Mazzocco & Devlin, 2008). The development of algebraic concepts such as ratio and rate of change is a prerequisite skill needed for algebra (National Mathematics Advisory Panel (NMAP), 2008) and these two concepts are rooted in part in the understanding of equivalence relationships. Consequently, fraction equivalence is an important concept in mathematics, as algebra is a "demonstrable gateway to later achievement" (NMAP, 2008, p. 5) in higher mathematics and a major factor in high school graduation rates (Achieve, Inc., 2006). Yet not much is known about the seemingly pervasive and debilitating difficulties children labeled as LD, in particular, experience understanding rational number equivalence.

The existing research literature outlines two possible sources of difficulty children with LD experience while learning about equivalence. A preponderance of the literature suggests domain-general cognitive factors (i.e., working memory, processing speed, or fluid reasoning, among others) impede children's use of developmentally appropriate problem solving strategies to solve mathematics tasks (Davis et al., 2009; Siegler, 2007). Yet, idiosyncratic strengths and weaknesses across varying cognitive domains are documented in the research (Compton, Fuchs, Fuchs, Lambert, and Hamlett, 2012); evidence of which cognitive factors, if any, impact children's understanding of rational number equivalence does not exist. Contrastingly,

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we might expect different children labeled as LD to exhibit all, some, or none of the cognitive factors associated with the disability as they learn about rational number equivalence (Vukovic, 2012). Thus, the identification of useful cognitive factors to study in intervention research that aims to improve children's concept of rational number equivalence is difficult.

There is also evidence which suggests that certain difficulties the children with LD experience while learning about rational number equivalence are associated with atypical understandings of the partitioning involved in part-whole rational number sub-constructs. Lewis (2014) found two such understandings in her work with adults: (a) *taking* (e.g., understanding shaded areas within part whole fraction representations as taken away; 3 out of 4 parts shaded would be interpreted as one-fourth) and (b) *halving* (e.g., interpreting a partition line as a fraction; the fraction ½ is interpreted as the partition line and not the resulting quantity). It is unclear if the same "atypical" understandings would be present in children; Lewis worked with adults. Interestingly, other research (Hunt & Empson, 2015) shows the misunderstandings documented by Lewis (2014) did not appear in elementary school children's work in the realm of equal sharing based problem tasks. Nonetheless, Lewis suggested the difficulties documented in her work were resistant to instruction and, further, that a mathematics LD might be due to a difference in how individuals with the disability attend to the representations involved in building mathematics conceptions. So, it is possible that children labeled as LD could experience difficulties in understanding equivalence concepts due to problematic interpretations of part to whole representations for rational numbers.

It is also possible that rational number sub-constructs that do not depend on children's interpretation of part-whole or partitioned representations may provide greater access to understanding equivalence for students with LD. Yet, part to whole approaches dominate many curriculums used in intervention research in special education (Butler, Miller, Crehan, Babbit, & Pierce, 2003; Jordan, Mercer, & Miller, 1999) and elsewhere, despite four decades of alternative approaches (c.r., Lamon, 2007). Few to no studies in special education intervention research explore the teaching and learning of rational number equivalence concepts facilitated by a different rational number sub-construct. Ratios are one such alternative and provide a direct link to the concept of equivalence- they are named as the rational number sub-construct most relevant to the teaching and learning of equivalence concepts and are thus appropriate in introducing concepts of equivalence (Behr, Post, & Silver, 1983). The following paragraphs provide a theoretical framework for using ratios to teach notions of equivalence.

Theoretical framework

Ratios can represent part-whole or part-part situations where the quantities are somehow related (Kieran, 1993; Marshall, 1993). Ratios introduce what Noelting (1980) describes as the between and within relationships of rational numbers. The between relationship analyzes the relationship between two or more ratios where an equivalent ratio is produced when multiplying the numerator and denominator by the same non-zero number. The within relationship examines the relationship between the numerator and denominator of a single ratio. Two ratios that are equivalent possess the same within, or unit, relation (Noelting, 1980). Although ambiguity still exists relating to the relationship between a ratio and a fraction (Clark, Berenson, & Cavey, 2003), researchers argue that "mathematics curriculum must not wait ... to advance multiplicative concepts, such as ratio" (Post et al., 1993, p. 331).

Researchers discuss several levels of progression regarding strategy usage and thinking as children progress toward understanding rational number equivalence through the ratio interpretation. Battista and Borrow (1995) suggest three phases of multiplicative understanding of equivalence situations understood through ratios: (a) conceptualize explicitly the linking action of two composite amounts; (b) understand multiplication/division and its role in the iteration process; and (c) abstract iterative processes and connect them to the meaning of multiplication and division (i.e. multiplicative understanding). Lamon (1993a) suggests multiple levels of corresponding strategies students use to reach multiplicative understandings involved with ratios: (a) avoiding (no interaction with the problem), (b) visual/additive (trial and error; additive linkages), (c) pattern building (oral or written patterns without understanding number relationships), (d) pre-proportional reasoning (pictures, charts, or manipulatives evidencing relative thinking), (e) qualitative proportional reasoning (ratio as unit/relative thinking/some numerical relation understandings), and (f) quantitative proportional reasoning (understanding of symbols, functional and scalar relationships). Researcher-teachers who wish to foster children's understanding of equivalence through the ratio interpretation might consider developmental progressions of children (i.e., those documented through thinking and strategy use described previously) and design tasks and supportive pedagogy based on an instructional trajectory of documented advances in children's conceptions (Daro, Mosher, & Cocoran, 2011).

To date, mathematics research in special education has not utilized developmental progressions situated in the ratio interpretation in designing teaching sequences aimed at improving understanding. One study was uncovered that looked at children's present levels of understanding specific to developmental progressions in ratio equivalence. Grobecker (1997) investigated 84 elementary aged children with and without LD and their ability to partition, unitize, and reason about fraction equivalence over multiple age groups. Four levels of understanding encompassed all solutions of children with and without LD similar to those found by Battista and Borrow (1995) and Lamon (1993a) with typically achieving populations. The researcher found that, with age, only children with LD did not advance beyond rudimentary thought structures to understand equivalence. Grobecker's data suggest differences in understanding between children with LDs and typically progressing children as regards multiplicative thought structures; these differences have been suggested in subsequent studies (e.g., Hecht et al., 2006; Mazzocco & Devlin, 2008). Yet, it remains an empirical question whether such understandings and development of the concept of rational number equivalence might be cultivated through teaching experiments situated in the ratio interpretations for children labeled as LD.