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A novel non-Lyapunov approach through artificial bee colony algorithm for detecting unstable periodic orbits with high orders ☆

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ABSTRACT

In this paper, a novel non-Lyapunov way is proposed to detect the unstable periodic orbits (UPOs) with high orders by a new artificial bee colony algorithm (ABC). And UPOs with high orders of nonlinear systems, are one of the most challenging problems of nonlinear science in both numerical computations and experimental measures. The proposed method maintains an effective searching mechanism with fine equilibrium between exploitation and exploration. To improve the performance for the optimums of the multi-model functions and to avoid the coincidences among the UPOs with different orders, we add the techniques as function stretching, deflecting and repulsion to ABC. The problems of detecting the UPOs are converted into a non-negative functions' minimization through a proper translation, which finds a UPO such that the objective function is minimized. Experiments to different high orders UPOs of 5 wellknown and widely used nonlinear maps indicate that the proposed algorithm is robust, by comparison of results through the ABC and quantum-behaved particle swarm optimization (QPSO), respectively. And it is effective even in cases where the Newton-family algorithms may not be applicable. Density of the orbits are discussed. Simulation results show that ABC is superior to QPSO, and it is a successful method in detecting the UPOs, with the advantages of fast convergence, high precision and robustness.

1. Introduction

In recent years, great interest emerged in swarm intelligence. For example, particle swarm optimization, ant colony optimization, bacterial foraging optimization algorithm, artificial bee colony (ABC) algorithm, and bee colony algorithms are typical methods of swarm intelligence (Akay & Karaboga, 2009a; Alatas, 2011; Gao, Li, & Tong, 2008; Gao, Qi, Yin, & Xiao, 2010c, 2010d; Gao & Tong, 2006; Guney & Onay, 2010; Mocholi, Jaen, Catala, & Navarro, 2010; Mullen, Monekosso, Barman, & Remagnino, 2009; Karaboga & Basturk, 2007; Thammano & Poolsamran, 2012; Yusup, Zain, & Hashim, 2012). ABC algorithm is one of the most recently introduced swarm-based algorithms, which models the intelligent foraging behavior of a honeybee swarm. Since Karaboga (2005) studied on ABC algorithm and its applications to real world-problems in 2005, ABC has been proven to be a better heuristic for global numerical optimization (Akay & Karaboga, 2009a; Akay & Karaboga, 2009b; Baig & Rashid, 2007; Gao, Qi, Yin, & Xiao, 2010a, Gao, Qi, Yin, & Xiao, 2010b, 2010c, 2010d; Karaboga & Basturk, 2008, 2009a; Karaboga & Akay, 2009b; Karaboga & Basturk, 2007).

The set of unstable periodic orbits (UPOs) can be thought of as the skeleton for the dynamics. Actually, one of the common characterizations of chaos is the positivity of topological entropy, which is related to the exponential growth rate of the number of UPOs embedded within the attractor as one enumerates the orbits

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by their lengths (Figueiredo & Chen, 1993; Katok & Hasselblatt, 2003; Ott, 1993; Ott, Grebogi, & Yorke, 1990; Zelinka, Celikovsky, Richter, & Chen, 2010, 2009). And Biham and Wenzel proposed a good method to find many UPOs especially for Hénon system (Biham & Wenzel, 1989). UPOs are also important tools in affecting the behaviors of dynamical systems. Furthermore, many dynamical averages, such as the natural measure, the Lyapunov exponents, the fractal dimensions, can be efficiently expressed in terms of a sum over the unstable periodic orbits (Dhamala & Lai, 1999; Gao, Gao, Li, Tong, & Lee, 2009; Ott, 1993; Pierson & Moss, 1995).

However, finding UPOs of nonlinear mapping is one of the most challenging problems of nonlinear science in both numerical computations and experimental measures, as the following reasons: UPOs' inner unstable nature and the analytic expressions for evaluating periodic orbits can be obtained only if the chaos system is the nonlinear polynomial of low degree and the period is low.

In most experimental simulations, a time series data is usually the only available information from a dynamical system to determine the positions of UPOs (Awrejcewicz, Dzyubak, & Grebogi, 2004; Davidchack & Lai, 1999; Schmelcher & Diakonos, 1997). Schmelcher and Diakonos have put an excellent algorithm (SD) (Schmelcher & Diakonos, 1997), which can be applied to detect higher periodic orbit and moreover it is globally converged. In addition, one modified version of SD has also been given to improve its converging rate (Bu, Wang, & Jiang, 2004; Davidchack & Lai, 1999) and etc. And even some theoretical analysis have been done to achieve the high orders UPOs, however their simulations are only given in the cases of UPOs with low orders (Albers & Sprott, 2006; Gluckman et al., 1997).

A basic scheme in detecting UPOs from dynamically reconstructed phase space (So et al., 1996; Takens, 1981) is to find recurrences of the iterated reconstructed map (Pierson & Moss, 1995). However, this method depended on the natural measure of the UPOs. An enhancement of the standard recurrence method was proposed later (So, Francis, Netoff, Gluckman, & Schiff, 1998). And these methods are not effective when the chaos systems are not differentiable. Then there are a couple of notable ones more recently: One is an adaptive control-based detection method proposed by Christini and Kaplan (2000).

Unlike the above Lyapunov methods, the other method is a totally new one, by a swarm intelligence (Gao, Lee, Li, Tong, & Lü, 2009; Gao et al., 2009; Gao et al., 2008; Gao, Qi, Balasingham, Yin, & Gao, 2012; Gao, Qi, Yin, & Xiao, 2012; Parsopoulos & Vrahatis, 2003) in non-Lyapunov way. It can succeed even when the nonexistence of derivatives or poorly behaved partial derivatives in the neighborhood of the fixed points. But the results are not so satisfied and need to be progressed.

The objective of this work is to present a novel simple but effective method to detect UPOs. In which, the UPOs are resolved by ABC. To improve the performance for the optimums of the multimodel functions and to avoid the coincidences among the UPOs with different orders, we add the techniques as stretching deflecting and repulsion to ABC. And the illustrative examples in different high order periods of 5 classical chaos systems system are discussed, respectively. These experiment results are much more better than quantum-behaved particle swarm optimization (QPSO) in Ref. (Gao et al., 2009; Gao et al., 2008).

The rest is organized as follows. Section 2 provides brief review for ABC. In Section 4, a proper mathematics model is introduced to transfer UPOs into a numerical optimization problems. In Section 5, firstly 5 famous nonlinear mappings are introduced, then simulations' comparisons are done to detect the different orders UPOs at different high orders by ABC and QPSO in Ref. (Gao et al., 2009) respectively, lastly the results are analyzed. Conclusions are summarized briefly in Section 6.

2. A novel artificial bee colony algorithm

2.1. Artificial bee colony algorithm

ABC algorithm has been inspired by the intelligent behavior of real honey bees (Akay & Karaboga, 2009a, 2009b; Baig & Rashid, 2007; Gao et al., 2010a, 2010b, 2010c, Gao, Qi, Yin, & Xiao, 2010d; Karaboga, 2005; Karaboga & Akay, 2009a, 2009b; Karaboga & Basturk, 2007, 2008). A bee swarm can provide different kinds of mechanisms used by the bees to adjust their flying trajectories. Compared with other population based methods, the significant difference is that the bee swarm contains miscellaneous groups, such as scouts, onlookers, foragers, etc. These lead to the emergence of collective intelligence of bee swarms consists of three essential components: food sources, employed foragers, and unemployed foragers, by a significant model for bee colony foraging behavior based on reaction–diffusion equations (Tereshko, 2000; Tereshko & Loengarov, 2005; Valery & Troy, 2002). Therefore, in typical ABC algorithm, the colony consists of three groups of bees: employed bees, onlookers and scouts, and two colony behaviors: recruitment to a food source and abandonment of a source (Akay & Karaboga, 2009a; Akbari, Mohammadia, & Ziarati, 2010; Alatas, 2010; Baig & Rashid, 2007; Karaboga, 2005; Karaboga & Akay, 2009a; Karaboga & Basturk, 2007; Gao et al., 2010a, 2010b, 2010c, 2010d). The position of a food source indicates a feasible solution; The nectar amount of a food source represents the fitness of the solution; The number of the employed bees or the onlooker bees is equal to the number of solutions in the population. And the onlookers and employed bees carry out the exploitation process in the search space, the scouts control the exploration process.

At the first step, ABC utilizes S_n (the size of employed bees or onlooker bees) D-dimensional individuals (food source positions), $\{x_i = (x_{i1}, \ldots, x_{iD}),\ (i = 1, \ldots, S_n)\}$ as initial population $P(C)$, $(C = 0)$.

In each cycle, the bee colony $P(C)$, $(C = 1, \ldots, K)$ perform the searching process: The employed bee retains the better position who has higher nectar amount in her memories according to the local information (current position) and the nectar amount (fitness value) of the new source (new solution). After all employed bees complete this process, they share the nectar information of the food sources and their position information (fitness) with the onlooker bees. Then the onlooker bee evaluates the nectar information taken from all employed bees and chooses a food source with a selection probability p_i related to its nectar amount.

$$p_i = \frac{fit_i}{\sum_{k=1}^{S_n} fit_k} \quad (1)$$

As in the case of the employed bee, she retains the better position who has higher nectar amount in her memories.

The ABC generates a candidate food position for each employed bee from the old one by:

$$v_{ij} = x_{ij} + \phi_{ij} \cdot (x_{ij} - x_{kj}) \quad (2)$$

where $k \in 1, \ldots, S_n$ $(k \neq i)$, $j \in 1, \ldots, D$ are randomly chosen indexes. ϕ_{ij} is a random number in $[-1, 1]$, which controls the production of neighbor food sources around x_{ij} and represents the comparison of two food positions visually by a bee. As in Eq. (2), the perturbation on x_{ij} gets decreased, when the difference between the parameters of the x_{ij} and x_{kj} decreases (Karaboga & Akay, 2009b). Thus, the step length is adaptively reduced as the search approaches the optimum solution. Actually, this operation results in two aspects: v_{ij} might exceed the predetermined limit, then it is set to its limit value; When the optimum is a local not a global optimum, the bee colony might not converge and flip–flop back and forth in local optimum area.