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Duplication externalities in an endogenous growth model with physical capital, human capital, and R&D

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ABSTRACT

This paper devises an endogenous growth model with physical capital, human capital and product variety. Differently to previous works, innovation is subject to externalities associated to the duplication of research effort, as well as to R&D spillovers. We provide conditions for the existence of a unique feasible steady-state equilibrium with positive long-run growth. For appropriate parameter values, the transitional dynamics of the model is represented by a two-dimensional stable manifold. Numerical simulations show that the incorporation of duplication externalities significantly increases the ability of the model to fit the observed data.

1. Introduction

Physical capital accumulation, knowledge formation, and R&D-based technological progress are considered the three main sources of growth. Although the bulk of the theoretical literature has treated them as alternative rather than complementary explanations, Arnold (2000) and Funke and Strulik (2000) (AFS henceforth) have combined them into an endogenous growth model with physical capital, human capital and R&D, whose equilibrium dynamics has been correctly analyzed by Gómez (2005). In the AFS model, invention of new ideas depends solely and linearly on effective time devoted to these activities. Based upon empirical evidence (e.g., del Barrio-Castro et al., 2002), Sequeira (forthcoming) incorporates R&D spillovers in innovation – a "standing on shoulders" effect (e.g., Jones, 1995a) – to the AFS model. However, the simulation results reported by Gómez (2005) – for the AFS model – and Sequeira (forthcoming) exhibit counterfactual highly oscillatory dynamics for variables as, e.g., education and innovation time for which history has shown a monotonic evolution (see, e.g. Jones, 2002).

This paper shows that adding an externality in R&D associated to the duplication and overlap of research effort – a "stepping on toes" effect (e.g., Jones, 1995a; Stokey, 1995) – to the Sequeira (forthcoming) model with R&D spillovers significantly increases its fit to data. Such duplication externality can be justified intuitively because the larger the number of people searching for ideas, the more likely it is that duplication of research would occur. In that case, doubling the number of researchers will less than double the number of unique ideas or discoveries. Empirical evidence of diminishing returns caused by duplicative research has been reported, e.g., by Kortum (1993). Lambson and Phillips (2007) found that the probability of duplication is not low for most industries. Griliches (1990) reported some evidence of diminishing returns found in the patent literature. Therefore, this duplication externality is also entirely plausible.

The purpose of this paper is twofold. First, we analyze the equilibrium dynamics of the model. Whereas the system that describes the dynamics of the AFS model has order three, Sequeira (forthcoming) shows that the introduction of R&D spillovers increases its order to four. We show that adding also duplication externalities further increases the order of the dynamic system to five. We provide conditions for the existence of a unique feasible steady-state equilibrium with positive long-run growth. We then analyze its (local) stability. The transition dynamics is represented by a two-dimensional stable manifold and, despite the complexity of the dynamic system, we provide a sufficient condition for stability. However, the instability outcome cannot be ruled out. Second, we present some numerical results showing that the introduction of duplication externalities significantly increases the ability of the model to generate realistic transition dynamics.

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 analyzes the balanced growth equilibrium. Section 4 presents some simulation results. Section 5 concludes.

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2. The model

2.1. Setup of the model

Consider a closed economy inhabited by a constant population, normalized to one, of identical individuals who derive utility from consumption, C, according to

$$\int_0^{\infty} e^{-\rho t}\left(C^{1-\theta}-1\right)/(1-\theta)dt, \quad \rho>0. \tag{1}$$

Individual's time, which is normalized to unity, can be devoted to production, u_P, education, u_E, or R&D, $u_I = 1 - u_P - u_E$. Human capital, H, is accumulated according to

$$\dot{H} = \xi u_E H, \quad \xi>0. \tag{2}$$

The budget constraint faced by the representative individual is

$$\dot{A} = rA + w(1-u_E)H-C, \tag{3}$$

where w is the wage rate per unit of employed human capital, and r is the return per unit of aggregate wealth A. Let g_x denote x's growth rate, $g_x = \dot{x}/x$. The individual maximizes her intertemporal utility (Eq. (1)), subject to the budget constraint (Eq. (3)) and the knowledge accumulation technology (Eq. (2)). The first order conditions for an interior solution yield

$$g_C = (r-\rho)/\theta, \tag{4}$$

$$r-g_w = \xi. \tag{5}$$

Output, Y, is produced with a Cobb–Douglas technology

$$Y = BK^{\beta}D^{\eta}(u_P H)^{1-\beta-\eta}, \quad B>0, \quad \beta>0, \quad \eta>0, \quad \beta+\eta<1, \tag{6}$$

where K is the stock of physical capital, and D is an index of intermediate goods, $D = \left(\int_0^n x(i)^{\alpha}di\right)^{1/\alpha}$, $0<\alpha<1$, where $x(i)$ is the amount used for each one of the n intermediate goods. The market for final goods is perfectly competitive and the price for final goods is normalized to one. Profit maximization delivers the factor demands

$$r = \beta Y/K, \tag{7}$$

$$w = (1-\beta-\eta)Y/(u_P H), \tag{8}$$

$$p(i) = \eta Y x(i)^{\alpha-1}/D^{\alpha}, \tag{9}$$

where $p(i)$ represents the price of intermediate i.

Invention of new intermediates is determined according to

$$\dot{n} = \bar{\delta}u_I H = \delta\left(\overline{u_I H}\right)^{\omega-1}n^{\phi}(u_I H), \quad \delta>0, \quad 0<\omega \text{b} 1, \quad 0\leq\phi \text{b} 1, \tag{10}$$

where $\overline{u_I H}$ represents average human capital devoted to innovation. This specification incorporates a duplication externality of research effort, as well as the potential for spillovers in R&D.[1]

There is monopolistic competition in the intermediate-goods sector, and an intermediate good costs one unit of Y to produce. Facing the price elasticity of demand for the intermediates $1/(1-\alpha)$, firms maximize operating profits, $\pi(i) = (p(i)-1)x(i)$, by charging a constant markup price $p(i) = 1/\alpha$. Since both technology and demand are the same for all intermediates, the equilibrium is symmetric: $x(i) = x$, $p(i) = p$. Hence, the quantity of intermediates employed is $xn = \alpha\eta Y$, firms' profits are

$$\pi = (1-\alpha)\eta Y/n, \tag{11}$$

and $D = xn^{1/\alpha} = n^{(1-\alpha)/\alpha}\alpha\eta Y$. Substituting this expression into Eq. (6) yields

$$Y^{1-\eta} = B(\alpha\eta)^{\eta}K^{\beta}n^{(1-\alpha)\eta/\alpha}(u_P H)^{1-\beta-\eta}. \tag{12}$$

An innovation υ is worth the present value of the stream of monopoly profits, $\upsilon(t) = \int_t^{\infty} e^{-r(\tau,t)}\pi(\tau)d\tau$, with $r(\tau,t) = \int_t^{\tau} r(s)ds$. Differentiating this expression with respect to time yields the no-arbitrage equation

$$g_{\upsilon} = r-\pi/\upsilon. \tag{13}$$

Finally, in an equilibrium with innovation, free-entry into R&D requires

$$w = \delta\left(\overline{u_I H}\right)^{\omega-1}n^{\phi}\upsilon. \tag{14}$$

2.2. Equilibrium dynamics

Henceforth we shall take into account that $\overline{u_I H} = u_I H$ in equilibrium. Let $\chi \equiv C/K$ denote the consumption to physical capital ratio, and $\psi \equiv H^{\omega}n^{\phi-1}$, the knowledge–ideas ratio. Physical capital and claims to innovative firms are the assets in the economy. Aggregate wealth is then $A = K + n\upsilon$. From Eqs. (3), (7)–(11) and (13) we can get the economy's resource constraint, $\dot{K} = (1-\alpha\eta)Y-C$, which can be expressed as

$$g_K = \frac{1-\alpha\eta}{\beta}r-\chi. \tag{15}$$

Some equations will be needed for solving the model. Log-differentiating Eqs. (7), (8), and (12), and eliminating g_Y, we get

$$g_r = -\frac{1-\beta-\eta}{\beta}g_w + \frac{(1-\alpha)\eta}{\alpha\beta}g_n, \tag{16}$$

$$g_{u_P} = -\frac{1-\eta}{\beta}g_w + \frac{(1-\alpha)\eta}{\alpha\beta}g_n + g_K-g_H. \tag{17}$$

Log-differentiating Eq. (10) yields

$$g_{g_n} = \omega\left(g_{u_I} + g_H\right)-(1-\phi)g_n. \tag{18}$$

Log-differentiating Eq. (14), and substituting g_{υ} from Eq. (13), π from Eq. (11), w from Eq. (8), and υ from Eq. (14), we get

$$g_w = r + (\omega-1)\left(g_{u_I} + g_H\right)-\frac{(1-\alpha)\eta}{(1-\beta-\eta)u_I}u_P g_n + \phi g_n. \tag{19}$$

The dynamics of the economy in terms of the variables r, χ, u_P, ψ and g_n is determined by

$$g_r = \frac{1-\beta-\eta}{\beta}(\xi-r) + \frac{(1-\alpha)\eta}{\alpha\beta}g_n, \tag{20}$$

$$g_{\chi} = \left(\frac{1}{\theta}-\frac{1-\alpha\eta}{\beta}\right)r + \chi-\frac{\rho}{\theta}, \tag{21}$$

$$g_{u_P} = \frac{(1-\alpha)\eta}{\beta}\left(r + \frac{g_n}{\alpha}\right)-\chi-\xi\left(1-u_P-\frac{g_n^{1/\omega}}{\delta^{1/\omega}\psi^{1/\omega}}\right) + \frac{(1-\eta)}{\beta}\xi, \tag{22}$$

$$g_{\psi} = \omega\xi\left(1-u_P-\frac{g_n^{1/\omega}}{\delta^{1/\omega}\psi^{1/\omega}}\right)-(1-\phi)g_n, \tag{23}$$

[1] Arnold (2000), Funke and Strulik (2000) and Gómez (2005) consider the case in which $\omega = 1$ and $\phi = 0$, and Sequeira (forthcoming), the case in which $\omega = 1$.