



Min–max game theory and nonstandard differential Riccati equations for abstract hyperbolic-like equations[☆]

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ABSTRACT

We consider the abstract dynamical framework of Lasiecka and Triggiani (2000) [1, Chapter 9], which models a large variety of mixed PDE problems (see specific classes in the Introduction) with boundary or point control, all defined on a smooth, bounded domain $\Omega \subset \mathbb{R}^n$, n arbitrary. This means that the input \rightarrow solution map is bounded on natural function spaces. We then study min–max game theory problem over a finite time horizon. The solution is expressed in terms of a (positive, self-adjoint) time-dependent Riccati operator, solution of a non-standard differential Riccati equation, which expresses the optimal qualities in pointwise feedback form. In concrete PDE problems, both control and deterministic disturbance may be applied on the boundary, or as a Dirac measure at a point. The observation operator has some smoothing properties.

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1. Mathematical setting and formulation of the min–max problem. Statement of main results

In this paper we return to the abstract dynamical setting of [1, Chapter 9] (“abstract hyperbolic-like equations”), which model a large variety of mixed PDE problems of hyperbolic-like type with either boundary or point control/disturbance, all defined on a smooth bounded domain $\Omega \subset \mathbb{R}^n$. They include: second-order hyperbolic equations and Schrödinger equations with control/disturbance in the Dirichlet B.C. [1, Sections 10.5 and 10.9]; non-symmetric, non-dissipative, first-order hyperbolic systems with boundary control/disturbance [1, Section 10.6]; Euler–Bernoulli and Kirchhoff plate equations with different types of boundary controls/disturbances [1, Sections 10.7; 10.8], even wave equations with Neumann boundary control/disturbance, however, only in one dimension [1, Section 9.8.4, p. 857]; as well as wave and Kirchhoff equations with point control/disturbance [1, Section 9.8], as well as systems of coupled PDE equations (wave and Kirchhoff equations; wave and structurally damped Euler–Bernoulli equations [1, Sections 9.10 and 9.11] arising in noise reduction models, with point control/disturbance). One may add elastic and thermoelastic dynamics with point control disturbance [2–5]. For such abstract dynamics, we then study a min–max game theory problem over a finite time horizon and with a smoothing observation operator. This is the perfect counterpart of the optimal control problem studied in [1, Chapter 9]. In the solution of this min–max problem that we provide here, all the optimal quantities – control, disturbance, state, observed state – are expressed explicitly in terms of the data of the problem via a time-dependent, positive definite Riccati operator, solution of a non-standard differential Riccati equation. For such hyperbolic-like dynamics, the corresponding min–max problem over infinite time interval was studied in [6,7] for the stable, respectively, unstable cases leading to an algebraic non-standard Riccati equation.

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For min–max problems for parabolic-like (analytic semigroup), we refer to [8, Section 9.8], [9,10].

For min–max problems in systems of coupled PDEs of different type (parabolic versus hyperbolic), we quote structural acoustic chambers [11]; a fluid–structure interaction model [12], as well as the abstract treatment in [13], see also [14].

Dynamical model (Setting in [8, Chapter 9], [15,16]). Let U (control), V (disturbance), Y (state) be separable Hilbert spaces. In this paper, we consider the following abstract dynamical system

$$\begin{cases} \dot{y}(t) = Ay(t) + Bu(t) + Gw(t) & \text{in } [\mathcal{D}(A^*)]' \\ y(0) = y_0 \in Y. \end{cases} \tag{1.1}$$

Here the function $u \in L_2(0, T; U)$ is the control and $w \in L_2(0, T; V)$ is a deterministic disturbance. The dynamics (1.1) is subject to the following assumptions, to be maintained throughout this paper:

- (A.1) $A : Y \supset \mathcal{D}(A) \rightarrow Y$ is the infinitesimal generator of a strongly continuous (s.c.) semigroup e^{At} on Y . Without loss of generality, as the dynamics (1.1) is studied here over a finite time interval $[0, T]$, $T < \infty$, at the price of replacing A with a suitable translation of A , we may assume that $A^{-1} \in \mathcal{L}(Y)$.
- (A.2) B and G are linear operators satisfying: $B \in \mathcal{L}(U; [\mathcal{D}(A^*)]')$ and $G \in \mathcal{L}(V; [\mathcal{D}(A^*)]')$, respectively. Equivalently, $A^{-1}B \in \mathcal{L}(U; Y)$, and $A^{-1}G \in \mathcal{L}(V; Y)$. Here, $[\mathcal{D}(A^*)]'$ is the dual space of the domain $\mathcal{D}(A^*)$, with respect to the pivot space Y . Thus, e^{At} can be extended as a s.c. semigroup on $[\mathcal{D}(A^*)]'$ as well.
- (A.3) The observation operator R is bounded:

$$R \in \mathcal{L}(Y; Z), \tag{1.2}$$

where Z (output space) is another Hilbert space.

- (A.4) The (closable) operators $B^*e^{A^*t}$ and $G^*e^{A^*t}$ can be extended (from $\mathcal{D}(A^*)$) to satisfy the ‘abstract trace regularity’ [1, p. 766], [17–21]:

$$B^*e^{A^*t} : \text{continuous } Y \rightarrow L_2(0, T; U); \quad G^*e^{A^*t} : \text{continuous } Y \rightarrow L_2(0, T; V), \tag{1.3a}$$

that is,

$$\int_0^T \|B^*e^{A^*t}y\|_U^2 dt \leq C_T \|y\|_Y^2; \quad \int_0^T \|G^*e^{A^*t}y\|_V^2 dt \leq C_T \|y\|_Y^2, \quad y \in Y \tag{1.3b}$$

(estimate (1.3b) is first checked for all $y \in \mathcal{D}(A^*)$ and then extended to all of Y).

- (A.5) The maps $R^*Re^{At}B$ and $R^*Re^{At}G$ can be extended as follows [1, p. 767], [21]:

$$R^*Re^{At}B : \text{continuous } U \rightarrow L_1(0, T; Y); \quad R^*Re^{At}G : \text{continuous } V \rightarrow L_1(0, T; Y), \tag{1.4a}$$

that is,

$$\int_0^T \|R^*Re^{At}Bu\|_Y dt \leq C_T \|u\|_U, \quad u \in U; \quad \int_0^T \|R^*Re^{At}Gw\|_Y dt \leq C_T \|w\|_V, \quad w \in V. \tag{1.4b}$$

Min–max game theory problem on $[0, T]$. For a fixed $0 < T < \infty$ and a fixed $\gamma > 0$, we associate with (1.1) the cost functional

$$J(u, w; y_0) = J(u, w, y(u, w); y_0) = \int_0^T \left[\|Ry(t)\|_Z^2 + \|u(t)\|_U^2 - \gamma^2 \|w(t)\|_V^2 \right] dt, \tag{1.5}$$

where $y(t) = y(t; y_0)$ is the solution of (1.1) due to $u(t)$ and $w(t)$. See below in (1.15a).

The aim of this paper is to study the following min–max game theory problem:

$$\sup_{w \in L_2(0, T; V)} \inf_{u \in L_2(0, T; U)} J(u, w; y_0), \tag{1.6}$$

where the infimum is taken over all $u \in L_2(0, T; U)$, for $w \in L_2(0, T; V)$ fixed, and the supremum is taken over all $w \in L_2(0, T; V)$. This problem, in addition to being of interest by itself, is known to be the state space formulation of the so called H^∞ robust stabilization problem; see [22,23]. In these references, this problem was introduced and stated in terms of the transfer function in the context of finite-dimensional theory [23]. The min–max game theory problem on the infinite time horizon, $[0, \infty]$ and thus leading to a non-standard algebraic Riccati equation, has been studied in [6,7,24,1]. In this paper, we are interested in a finite time setting, $[0, T]$, and the corresponding non-standard differential Riccati equation.

Dual versions of (A.4), (A.5). By duality in (A.4), we obtain the following relations [8, p. 767]. First, on $B^*e^{A^*t}$:

- (A.4*) The map L_0 satisfies [18], [1,21, Chapter 7, Theorem 7.2.1]

$$(L_0u)(t) = \int_0^t e^{A(t-\tau)} Bu(\tau) d\tau : \text{continuous } L_2(0, T; U) \rightarrow C([0, T]; Y), \tag{1.7}$$

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