



Evolutionary game theory on measure spaces: Well-posedness

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ABSTRACT

An attempt is made to find a comprehensive mathematical framework in which to investigate the problems of well-posedness and asymptotic analysis for fully nonlinear evolutionary game theoretic models. The model should be rich enough to include all classical nonlinearities, e.g., Beverton–Holt or Ricker type. For several such models formulated on the space of integrable functions, it is known that as the variance of the payoff kernel becomes small the solution converges in the long term to a Dirac measure centered at the fittest strategy; thus the limit of the solution is not in the state space of integrable functions. Starting with the replicator–mutator equation and a generalized logistic equation as bases, a general model is formulated as a dynamical system on the state space of finite signed measures. Well-posedness is established, and then it is shown that by choosing appropriate payoff kernels this model includes all classical density models, both selection and mutation, and discrete and continuous strategy (trait) spaces.

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1. Introduction

Evolutionary game theory (EGT) is the creation and study of mathematical models that describe how the strategy profile in games change over time due to mutation and selection (replication). In this paper we address the problem of finding a comprehensive mathematical framework suitable for studying the problems of well-posedness and long-term solution behavior for fully nonlinear evolutionary game theoretic models. We form a unified theory for evolutionary game theory as a dynamical system on the state space of finite signed Borel measures under the weak star topology. In this theory, we unify the discrete and continuous strategy (trait) spaces and the pure replicator and replicator–mutator dynamics under one model.

A natural question to ask is why the formulation of a dynamical system on the state space of finite signed Borel measures under the weak star topology? Why isn't the existing mathematical machinery adequate? The next two examples will illustrate the need for such a formulation. First, we consider the following EGT model of generalized logistic growth with pure selection (i.e., strategies replicate themselves exactly and no mutation occurs) which was developed and analyzed in [1]:

$$\frac{d}{dt}x(t, q) = x(t, q)(q_1 - q_2X(t)), \quad (1)$$

where $X(t) = \int_Q x(t, q)dq$ is the total population, $Q \subset \text{int}(\mathbb{R}_+^2)$ is compact and the state space is the set of continuous real valued functions $C(Q)$. Each $q = (q_1, q_2) \in Q$ is a two tuple where q_1 is an intrinsic replication rate and q_2 is an intrinsic mortality rate. The solution to this model converges to a Dirac mass centered at the fittest q -class. This is the class

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with the highest birth to death ratio $\frac{q_1}{q_2}$, and this convergence is in a topology called *weak** (point wise convergence of functions) [1]. However, this Dirac limit is not in the state space as it is not a continuous function. It is a measure. Thus, under this formulation one cannot treat this Dirac mass as an equilibrium (a constant) solution and hence the study of linear stability analysis is not possible. Other examples for models developed on classical state spaces such as $L^1(X, \mu)$ that demonstrate the emergence of Dirac measures in the asymptotic limit from smooth initial densities are given in [2,1,3–8]. In particular, how the measures arise naturally in a biological and adaptive dynamics environment is illustrated quite well in [7, Chapter 2]. These examples show that the chosen state space for formulating such selection–mutation models must contain densities and Dirac masses and the topology used must contain the ability to demonstrate convergence of densities to Dirac masses.

The first example above assumes a continuous strategy space Q and hence the model solution is sought among density functions denoted by $x(t, q)$. Our second example, is the classic discrete EGT model known as the replicator–mutator equation (in this model the strategy space is assumed to be discrete). In [9, p. 273] it is given as

$$\frac{d}{dt}x_i(t) = \sum_{j=1}^n x_j(t)f_j(\vec{x}(t))Q_{ij} - \phi(\vec{x}(t))x_i(t) \quad (2)$$

where $\vec{x}(t) = (x_1(t), x_2(t), \dots, x_n(t))$ is a vector consisting of n classes each of size $x_i(t)$, and Q_{ij} is the payoff kernel, i.e., Q_{ij} is the proportion of the j -class that mutates into the i -class and ϕ is a weighted (average) fitness. The author states that the language equation (replicator–mutator equation) is a unifying description of deterministic evolutionary dynamics. He further states that the replicator–mutator equation is used to describe the dynamics of complex adaptive systems in population dynamics, biochemistry and models of language acquisition.

Equation (1) and a version of (2) are special cases of a more general measure-valued model that we present in this paper. In particular, with the discrete model (2) if we assume that f_j and ϕ are functions of the total population $X = \sum_{j=1}^n x_j$ then this model can be obtained by choosing the proper initial condition composed of a linear combination of Dirac masses and the proper replication–mutation kernel which is also composed of a linear combination of Dirac masses. The example of the pure selection density model given in (1) can be realized from the measure-valued model by choosing an absolutely continuous initial measure and a continuous family of Dirac measures for the selection–mutation kernel (which represents the pure replication case). Thus, these density and discrete models can be unified under this formulation. Furthermore, our new theory combines both the pure replicator and replicator–mutator dynamics in a continuous manner. By this we mean that as the mutations get smaller and smaller the replicator–mutator model will approach the pure replicator model. This is possible because our mutation kernels are allowed to be (family of) measures as well. This presents a serious difficulty in the analysis which requires the development of some technical tools in studying the well-posedness of the new model.

Many researchers have recently devoted their attention to the study of such EGT models (e.g. [2,1,3,4,10–13]). To date almost all EGT models are formulated as density models [1,3,4,12,13] with linear mutation term. There are several formulations of pure selection or replicator equation dynamics on measure spaces [2,14,15]. The recent formulations of selection–mutation balance equations on the probability measures by [16,17] are novel constructions. These models describe the aging of an infinite population as a process of accumulation of mutations in a genotype. The dynamical equation which describes the system is of Kimura–Maruyama type. Thus far in selection–mutation studies the mutation process has been modeled using two different approaches: (1) a diffusion type operator [5,13]; (2) an integral type operator that makes use of a mutation kernel [2–4,16,17]. Here we focus on the second approach for modeling mutation.

Perhaps the work most related to the one presented here is that in [2]. In that paper, the authors considered a pure selection model with density dependent birth and mortality function and a 2-dimensional trait space on the space of finite signed measures. They discussed existence–uniqueness of solutions and studied the long term behavior of the model. Here, we generalize the results in that paper in several directions. Most salient is the fact that the present paper is one in evolutionary game theory; hence the applications are possibly other than population biology. In particular, in the present paper we construct a (measure valued) EGT model. This is an ordered triple (Q, μ, F) subject to

$$\frac{d}{dt}\mu(t)(E) = F(\mu(t)(Q))(E), \quad \text{for every } E \in \mathcal{B}(Q). \quad (3)$$

Here Q is the strategy (metric) space, $\mathcal{B}(Q)$ are the Borel sets on Q , $\mu(t)$ is a time dependent family of finite signed Borel measures on Q and F is a density dependent vector field such that μ and F satisfy Eq. (3). The main contributions of the present work are as follows: (1) we establish well-posedness of the new measure-valued dynamical system; (2) we are able to combine models that consider both discrete and continuous parameter spaces under this formulation; no separate machinery is needed for each; (3) we are able to include both selection and mutation in one model because our setup allows for choosing the mutation to be a family of measures; (4) unlike the linear mutation term commonly used in the literature, we allow for nonlinear (density dependent) mutation term that contains all classical nonlinearities, e.g., Ricker, Beverton–Holt, Logistic; (5) unlike the one or two dimensional strategy spaces used in the literature, we allow for a strategy space Q that is possibly infinite dimensional. In particular, we assume that Q is a compact complete separable metric space, i.e., a compact Polish space.

This paper is organized as follows. In Section 2 we demonstrate how to proceed from a density model to a measure valued one and we formulate the model on the (natural) space of measures. In Section 3 we establish the well-posedness of

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