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Two dimensional noncausal AR-ARCH model: Stationary conditions, parameter estimation and its application to anomaly detection

Saman Mousazadeh*, Israel Cohen

Department of Electrical Engineering, Technion-Israel Institute of Technology, Technion City, Haifa 32000, Israel

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ABSTRACT

Image anomaly detection is the process of extracting a small number of clustered pixels which are different from the background. The type of image, its characteristics and the type of anomalies depend on the application at hand. In this paper, we introduce a new statistical model called noncausal autoregressive–autoregressive conditional heteroscedasticity (AR-ARCH) model for background in sonar images. Based on this background model, we propose a novel anomaly detection technique in sonar images. This new statistical model (i.e. noncausal ARCH) is an extension of the conventional ARCH model. We provide sufficient stationarity conditions and develop a computationally efficient method for estimating the model parameters which reduces to solving two sets of linear equations. We show that this estimator is asymptotically consistent. Using matched subspace detector (MSD) along with noncausal AR-ARCH modeling of the background in the wavelet domain, we propose an anomaly detection algorithm for sonar images, which is computationally efficient and less dependent on the image orientation. Simulation results demonstrate the performance of the proposed parameter estimation and the anomaly detection algorithm.

1. Introduction

Image anomaly detection refers to the problem of finding regions in an image which do not conform to the expected behavior. The characteristic of the image and the type of anomalies are application dependent. Anomaly detection has a wide range of applications, such as silicon wafer defect detection [1,2], medical imaging [3] and sea-mine detection in side-scan sonar images [4] just to name a few. In sea-mine detection, lethal targets must be detected with nearly 100% reliability. False detections are not disastrous but might slow down the demining process. Every anomaly detection algorithm consists of some or all of the following stages: selection of an appropriate feature space; choosing an appropriate statistical model which represents the image background and selection of a detection algorithm.

A proper selection of a feature space, which allows distinction of anomalies from the background, is an important part of an anomaly detection algorithm. Features can be extracted from the image pixels themselves or from the image after passing through a transform. Kazantsev et al. [5] introduced a feature space based on two circular concentric windows W_1 and W_2 with radii R_1 and R_2 ($R_1 < R_2$), respectively. A similar approach was taken by Schweizer and Moura [6], where two concentric rectangles serve as the moving window. In these methods the features are extracted directly from the image itself. Features can be also extracted from the image in a transformed domain. Laine et al. [7] used a dyadic wavelet transform in mammography to emphasize mammographic features while reducing the noise. Strickland and Hahn [8] used an undecimated wavelet transform for detection of Gaussian objects in Markov noise. Xia et al. [9] used the

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* Corresponding author.
E-mail addresses: smousazadeh@gmail.com, smzadeh@tx.technion.ac.il (S. Mousazadeh), icohen@ee.technion.ac.il (I. Cohen).

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wavelet transform to insert an undetectable watermark into digital imagery. Noiboar and Cohen [4] used undecimated discrete wavelet transform (DWT) for anomaly detection in sonar images.

Once we have extracted our feature either from the image itself or from the transformed image, we must find a statistical model to describe the background in the selected feature space domain. A survey of current literature shows that the most popular models for the background are Gaussian and its extensions. These models are used mostly because of their mathematical tractability. Ashton [10] performed a sub-pixel anomaly detection in multispectral infrared imagery using a Gaussian distribution. Stein et al. [11] used a Gaussian mixture model (GMM) for modeling hyper-spectral imagery. Other extensions such as linear mixing model (LMM) and Gauss–Markov random field (GMRF) are used by several authors for modeling the background. A review of multi-resolution Markov models for signal and image processing can be found in [12]. A survey of results on the structure of two dimensional wide-sense stationary processes with special emphasis on finite order models can be found in [13]. The correlation structure of spatial linear and spatial moving average processes defined on a square lattice has been reviewed by Marc [14].

The generalized autoregressive conditional heteroscedasticity (GARCH) model was first introduced by Bollerslev [15] as an extension of the autoregressive conditional heteroscedasticity (ARCH) model developed by Engle [16] to model econometric data. Since then, many researchers have extended and used these models in several speech and image processing applications. Cohen [17] modeled the speech signal in the short time Fourier transform (STFT) domain as a complex GARCH process and used this model for speech enhancement. AR-GARCH model was utilized for modeling speech signal in the time domain and for developing voice activity detection (VAD) algorithms [18,19]. Abdolahi and Amindavar [20] used the parameters of the GARCH model for speech recognition in Persian isolated digits. Amirmazlaghani et al. [21] used two dimensional GARCH model for speckle suppression in SAR images. Two dimensional GARCH model is also used in image denoising [22]. Noiboar and Cohen [4] used *causal* GARCH model for anomaly detection in sonar images. The causality assumption incorporated into the GARCH model in [4] is unnatural for images. Developing a non-causal statistical model may lead to an improvement in detection performance by reducing the dependency of the detection procedure on image orientation.

In [23], we presented an anomaly detection method in sonar images based on noncausal AR-ARCH model. The background of the sonar image in the wavelet domain was modeled by a noncausal AR-ARCH model. Matched subspace detector (MSD) was used for detecting the anomaly in the image. In MSD it is assumed that the anomalies are within a subspace. This subspace is assumed to be known or can be estimated using training data.

In this paper, we provide sufficient stationary conditions for the model and propose an effective least squares method for estimating the model parameters. This estimator, which is shown to be asymptotically consistent, is obtained by solving two sets of linear equations and have a closed-form expression. We also present the detection algorithm in more details. This algorithm is based on the noncausal AR-ARCH modeling of the background and MSD. The rest of the paper is organized as follows. In Section 2, we introduce a two dimensional noncausal ARCH model, provide sufficient stationary conditions and develop a novel technique for estimating the parameters of this model. We also show in the Appendix that this estimator is asymptotically consistent. In Section 3, we introduce our anomaly detection algorithm, which is based on noncausal autoregressive ARCH model and MSD. This section is a detailed description of the method previously proposed by the authors in [23]. In Section 4, the performances of the parameter estimation and anomaly detection are evaluated using simulations. We conclude the paper in Section 5.

2. Noncausal ARCH model

2.1. Two dimensional noncausal ARCH model and its stationary conditions

We define a two dimensional noncausal ARCH(p,q) model as follows:

$$x(t_1,t_2)=\sigma(t_1,t_2)\varepsilon(t_1,t_2), \tag{1}$$

$$\sigma^2(t_1,t_2)=c_0+\sum_{i=-p}^{p}\sum_{j=0}^{q}a_{i,j}(x^2(t_1-i,t_2-j)+x^2(t_1+i,t_2+j)), \tag{2}$$

$$c_0>0 \quad a_{i,j}\geq 0, \tag{3}$$

where c_0 and a_{ij}'s are the two dimensional noncausal ARCH parameters, $\forall i\geq 0$ $a_{i,0}=0$, p and q are the model orders in the horizontal and vertical directions, respectively. $\varepsilon(t_1,t_2)$'s are zero mean independent identically distributed (IID) random variables with unit variance and $\sigma^2(t_1,t_2)$'s are called the conditional variances. The constraints (3) guarantee the positiveness of σ^2. These equations simply state that each pixel in the image (i.e. $x(t_1,t_2)$) is a random variable whose conditional variance (conditioned on the neighboring pixels) is a weighted sum of the squared value of the neighboring pixels. The neighborhood is determined by p and q. This definition is explained graphically in Fig. 1. This figure shows that the conditional variance of the centered pixel is a weighted mean of the squared values of the neighboring pixels where the weights are symmetric (i.e. pixels dotted with the same color have the same weight). This condition is applied to the definition of the model in order to make it identifiable. More specifically, since our parameter estimation method is based on the LS method proposed in [24] for estimating the parameters of the noncausal AR model, and a noncausal AR model is identifiable by the LS method if and only if the weights are symmetric, we ought to assume that the weights are symmetric. Two dimensional noncausal AR model represents the gray-scale level at a specific pixel, as a linear combination of the gray-scale levels of neighboring pixels and an additive white noise. This model has been used in many applications in image