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Tensor-based anomaly detection: An interdisciplinary survey

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A B S T R A C T

Traditional spectral-based methods such as PCA are popular for anomaly detection in a variety of problems and domains. However, if data includes tensor (multiway) structure (e.g. space-time-measurements), some meaningful anomalies may remain invisible with these methods. Although tensor-based anomaly detection (TAD) has been applied within a variety of disciplines over the last twenty years, it is not yet recognized as a formal category in anomaly detection. This survey aims to highlight the potential of tensor-based techniques as a novel approach for detection and identification of abnormalities and failures. We survey the interdisciplinary works in which TAD is reported and characterize the learning strategies, methods and applications; extract the important open issues in TAD and provide the corresponding existing solutions according to the state-of-the-art.

1. Introduction

Those patterns in data that do not conform to expected behavior are called *anomalies* and the process of detection of such patterns is known as *anomaly detection* [1]. Anomaly detection is an essential component of many safety, monitoring and surveillance systems. The reason is that it uncovers significant and critical facts about the system's behavior that leads to prevention of further escalation and losses. Plenty of methods have been developed during the last two decades for anomaly detection in different domains, the majority of which are covered in the survey paper [1]. One group of methods that is mentioned in this survey is spectral methods. These approaches attempt to project high dimensional data onto a lower subspace in which anomalies can be identified more easily. The main assumption of these techniques is that normal and abnormal instances appear significantly different in the projected subspace [1]. However, in many real-world applications we deal with data with tensor (multiway) structure which unfortunately is widely ignored. In such circumstances, anomalies may remain invisible with the matrix-based spectral methods. Besides, ignoring the tensor structure in data can cause some problems and result in wrong results. As an example some real failure case studies of matrix-based solutions and superiority of tensor-based solutions over them are listed in Table 1 which can manifest how much tensors are required for anomaly detection.

Although authors in [1] discuss the matrix methods in their survey, they exclude tensors and their applications in anomaly detection. This is while over the last twenty years, since the work of Nomikos and MacGregor [12], research related to tensor-based anomaly detection (TAD) has been exponentially growing. Furthermore, many methods have been developed in multiple disciplines from chemometrics and environmental monitoring to signal processing and data mining. Despite the popularity of this research area (though with different terminologies), no comprehensive survey on TAD is yet available. The most probable reason is that the TAD belongs to wide scopes and spans across different research fields.

Our main objective in this survey is to bridge the gap between two popular research areas of anomaly detection and tensors. We study the literature from all major disciplines where tensors are frequently applied and classify the contributions related to TAD based on some factors such as applications, learning types, methods and evaluation metrics. Moreover, we identify and classify the important issues and proposed solutions in TAD research. We follow a motivational strategy in this survey, in the sense that we do not limit ourselves introducing only techniques that are already applied for anomaly detection. Rather, we include those methods that are used in the close applications, such as classification, regression and forecasting that may show a great potential for anomaly detection. Therefore, this survey can be regarded as a comprehensive complement for Section 9 of [1] and from the tensor point of view it can be considered as a focused complement for applications of tensors in data mining, i.e. survey paper in [13]. Our assumption is that the reader is familiar with basic concepts

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Table 1
Some empirical evidences in the literature indicating the superiority of tensor-based solutions over matrix solutions.

Study	Tensor method	Matrix method	Matrix method's reported problem
[2]	Tucker3	PCA	Difficult interpretation of score plots
[3]	PARAFAC	PCA	Difficult interpretation of score plots
[4]	Non-negative Multiway PCA	PCA	Lower classification accuracy
[5]	Incremental Tensor subpsace learning	PCA	Lower tracking performance
[6]	Multiway PCA	PCA	Higher error rate in damage detection
[7]	Multiway PCA	PCA	Lower recognition accuracy
[8]	HOSVD	SVD	Higher prediction error
[9]	Tucker3	SVD	SVD fails on modeling tensor structured data
[10]	PARAFAC	PCA	Loss of multiway linkages plus over-fitting
[11]	PARAFAC	PCA	PCA fails to identify the right variance

Table 2
Tensor-based anomaly detection examples.

Domain	Typical tensor	Application	Ref.
Process control	Batch × Measurements × Time	Detection of faulty batches	[12]
Environment	Variables × Site × Time	Detection of spatiotemporal source of pollution	[18]
Video surveillance	ImgRow × ImgCol × Time	Abnormal event/objects discovery	[19]
Network security	OriginIP × DestIP × Time	Abnormal traffic discovery	[16]
Social networks	Person × Person × Time	Event detection	[20]
Text-based systems	Actor × Keyword × Time	Event detection	[21]
Neuroscience	Frequency × Channel × Time	Seizure recognition	[22]
Remote sensing	ImgRow × ImgCol × Wavelength	Target detection	[23]
Sensors	Measurements × Location × Time	Anomaly detection	[15]
Transportation	Origin × Destination × Time	Detection of urban traffic problems	[24]
Metallurgy Eng.	Coils × PSD × Frequency	Fault detection in hot strip mill	[25]
Civil structures	Location × Time × Frequency	Detection of damages in civil structures	[26]
Mechanical systems	Experiment × Sensor × Time	Damage detection in aircraft wing flap	[6]
Power systems	Experiments × Variables × Time	Detection of voltage sags	[27]
Medical diagnosis	Medication × Patient × Diagnosis	Heart failure prediction	[28]
Epidemiology	Space × Time × Indicators	Disease outbreak prediction	[29]
Seismology	Location × Time × Frequency	Predicting earthquake ground motion	[30]
Criminology	Lng × Lat × Time × Indicators	Crime occurrence forecasting	[31]

in anomaly detection and tensor decomposition (or tensorial learning). For this reason, we omit explanation of the straightforward concepts related to tensor decomposition, anomaly detection and spectral-based anomaly detection. Instead, we refer the reader to the recent surveys about anomaly detection [1] and tensor decomposition [13,14] that adequately cover essential technical materials for understanding the current review.

The article is organized as follows. In Section 2, we introduce the history of TAD and its applications. Section 3 presents learning methods for TAD. Section 4 discusses the techniques for tensor decomposition. Section 5 outlines the issues in TAD along with the corresponding solutions. In Section 6 we discuss the evaluation metrics used in TAD and introduce the available software for tensor analysis. Section 7 concludes the survey.

2. History and applications

A tensor is a geometric object used in mathematics and physics for extension of concepts such as scalars, vectors and matrices to higher dimensions. The origin of the word "tensor" is the Latin *tendere* "to stretch" firstly appeared in anatomy in the seventeenth century to denote muscle's stretch. It was later used in mid-eighteenth-century by William Hamilton to describe some concepts in quaternion algebra. Tensor calculus, which comes closer to the word's current meaning, was introduced in 1900 by Italian mathematician Gregorio Ricci-Curbastro and his doctoral student Tullio Levi-Civita. In 1915, tensor was used by Albert Einstein in general relativity theory for explaining geometric and causal structure of space-time and definition of concepts such as distance, volume, curvature, angle, future and past. The first principles of tensor decomposition [14] were founded by American mathematician Frank Hitchcock in 1927. Complex and multiway structure of human behaviors was probably the first motivation for use of tensors in data analysis. Psychologists such as Raymond Cattell, Ledyard Tucker and Richard Harshman were pioneers in extending tensor decomposition applications in psychology during three decades from 1940s to 1970s. In 1981, tensor decomposition was introduced by Appellof and Davidson to the Chemometrics community. The first applications of tensors in anomaly detection appeared in this community almost a decade later. The work of Nomikos and MacGregor [12] about multi-way batch monitoring was a pioneer in motivating tensor (multiway) methods in the monitoring and fault detection problems. The modern application of tensors in anomaly detection appeared a decade ago in a series of articles from Jimeng Sun and colleagues [15–17] who had a major contribution to the growth of TAD research. Nowadays, TAD's application has been widespread in wider areas, including environmental monitoring, video surveillance, network security, social networks, text-based systems, neuroscience, remote sensing, engineering and other domains. In the following, some of these applications are discussed in more detail (See Table 2 for summary).

2.1. Process control

The first footprint of tensor(multiway) methods as earlier mentioned can be seen in the monitoring of batch processes. The common objective in operating batch processes is to achieve value-added products of high-quality with competitive prices. The goal of the batch process analysis is to understand the major sources of batch-to-batch variations [12], real-time detection of faulty batches and use it to improve the operation policies.

Tensors are very popular monitoring techniques in production of chemicals and other manufacturing applications. Examples are polymerization processes [32–35], semiconductor etching process