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Size functions for comparing 3D models

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Abstract

This paper proposes an original framework to use size functions in the 3D context. Size functions are a mathematical tool, that have already shown its effectiveness for image retrieval and classification. They are introduced here for the first time to discriminate among 3D objects represented by triangle meshes, through the proposal of a method for defining size graphs independently of the underlying triangulation. We first derive a skeletal signature, which guarantees the topological coding and the geometric description of an object surface, then this signature is used as a size graph to compute discrete size functions. The attractive feature of size functions is that it readily gives a similarity measure between shapes. The result is the introduction of a new technique for 3D model retrieval, devised to capture both local and global properties of a shape. Finally, we demonstrate the potential of our approach in a set of experiments, and discuss the results with respect to existing techniques.

Keywords: Skeletal graph; Size graph; Size function; Matching distance; Shape comparison; 3D shape retrieval

1. Introduction

Shape comparison plays a fundamental role in the computer vision and computer graphics fields. In the last decade, the advances in modeling, digitizing and visualizing 3D shapes have led to an explosion in the number of available 3D models, both on the Internet and in domain-specific databases. Examples are digital repositories recording cultural heritage [1] or archives of structural data of biological macromolecules [2]. Whilst it has become relatively easy to generate 3D information and interact with the geometry of shapes, it is harder to structure, filter, organize and retrieve it. How to find and interpret 3D content has become the key issue, leading to the development of the first experimental search engines for 3D shapes, such as the 3D model search engine at Princeton University [3] or the 3D retrieval engine at Utrecht University [4].

From a high-level perspective, the main components of a retrieval system for 3D or 2D visual media are similar: a feature extraction module and an indexing system, that usually works off-line, and a matching module, that extracts online the most relevant items out of a collection, according to some metrics defined on the feature space. Human perception has been widely studied and supported a significant amount of work in computer vision related to the analysis and recognition of images [5,6]. The main differences between 2D pixel-based and 3D vector-based contexts arise in the feature extraction step and, unfortunately, most of the methods developed for images do not generalize directly to 3D shapes. This is mainly due to the different nature of the content: descriptors used for 2D images are concerned with color, textures and properties that capture geometric details of the shapes segmented in the image. Most notably, feature extraction for image retrieval is intrinsically affected by the possible presence of information, which is only accidental in the image or due to occlusion and/or perspective distortion. On the other hand, the boundary of 3D models is represented in vector form and therefore does not need to be segmented from a background. Possessing the complete geometry of 3D models allows for more effective and reliable search tools, although the intrinsic complexity of 3D shapes still makes the understanding of their content an arduous problem.

In this context, we are working for the development of a 3D search platform, that provides a rich and flexible set of descriptors within the same framework. Shape properties and

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their relevance to the search are obviously dependent on the user and context, meaning that we are able to index an object with different features according to the context in which we are asked to describe it. We aim to provide a modular system for shape comparison and retrieval, which allows users to fit a descriptor to the shape idea they have in mind. *Size theory* offers the theoretical support for the development of such type of framework. In particular, in this paper we introduce a new technique for 3D shape retrieval, which builds on *size functions*, a mathematical descriptor, which can readily and efficiently be used to establish a similarity measure between shapes.

The theory of size functions has been developed since the beginning of the 1990s in order to get a new geometric-topological approach to shape discrimination [7,8]. The idea is to analyze a given shape by exploring the growth of a topological space S associated to the shape, according to the increasing values of a real function φ defined on it. Intuitively, size functions code the topological evolution of S counting the number of connected components which remain disconnected passing from a lower level set of S to another. Since the growth of S is driven by the real function φ, size functions encode the geometrical properties of S captured by φ in the topological evolution of S. An introductory presentation can be found in Refs. [9,10]. In more recent years, a similar approach has been applied in a homological setting, independently leading to the introduction of persistent homology [11,12] and the definition of the Morse homology descriptor [13].

The most interesting aspect of size theory is that we can obtain a rich set of descriptors by simply changing the real function φ, while keeping the same matching framework. Another interesting aspect is the possibility to apply the theory not only to the shape itself, but also to an auxiliary space, associated to the shape, and enhancing its relevant characteristics [14]. This idea is exploited in the present paper.

Size functions have been extensively applied to content-based image retrieval and classification (see for example [8,15,16]), proving their effectiveness for describing and comparing objects. However, despite the general validity of the mathematical theory of size functions in spaces of any dimension, their application to 3D objects has never been performed before, and is not straightforward, as discussed in Section 3. The aim of this paper is to start exploiting and enhancing the potential of size functions for 3D shape comparison. The result is the definition of an effective technique for 3D shape description and retrieval, taking into account structure, topology and geometry. This paper is an evolution of the short paper presented in June 2006 at the Eurographics Symposium on Geometry Processing [17].

1.1. Paper contribution

The major contribution of the paper is the introduction of a method to apply size functions to the 3D shape matching context. The key idea is to associate a topological space S to a 3D shape M, where S is simpler than M in terms of dimensionality, while still preserves the main geometrical, structural and topological properties of the shape. This lower dimensional space S, or compact representation of M, is the input for the computation of size functions and the successive comparison process. We show the results obtained by using a 1D topological structure, called the *skeletal graph*, as an auxiliary space to study a 3D shape M.

The basic steps of the proposed method are:

- The definition of a suitable *skeletal graph* to describe 3D shapes, based on the construction of a centerline skeleton.
- The definition of a set of *measuring functions* on the skeletal graphs, which capture quantitative attributes of the shape.
- The computation of the size functions of the attributed skeletal graphs.
- The evaluation of the similarity between two models, through a suitable distance between their size functions.

Since the computation of size functions on 1D structures is very efficient, it is possible to use a varied set of attributed skeletal graphs—called *size graphs*—each reflecting the description of M with respect to some criteria.

For the construction of the skeletal graph, we adopt a schema grounded in Morse theory, based on the computation of the level sets of different real-valued functions (referred to in what follows as *mapping functions*) computed on the shape. The level sets drive the construction of the 1D centerline skeleton of the shape M, and also support the computation of several geometric attributes (referred to as *measuring functions*). The possibility of computing level sets of different mapping functions and the analysis of different geometric attributes allow us to build a set of descriptors, that capture a variety of shape properties, making our retrieval method a general and flexible framework. In other words, the system modularity allows the user to fit a descriptor to the specific nature of the object to be found, through the choice of the real functions (namely, mapping and measuring functions).

Moreover, since our size graphs are a skeletal representation of the shape, another interesting contribution of the paper is the introduction of a distance function for graph representations of 3D models. In this case, the distance between structural representations does not rely on the computation of exact or approximate (sub-)graph isomorphisms. Indeed, we replace the problem of graph matching (*NP*-hard in the classical formulation) with the simpler algebraic comparison of the size functions associated with the graphs.

The remainder of the paper is organized as follows. In Section 2 we briefly overview existing techniques for 3D object retrieval. After introducing size functions and discussing their use for 3D shapes in Section 3, we detail our approach to obtain informative size functions from 3D models in Section 4. The shape comparison process is outlined in Section 5. Relationships with related approaches in the literature are discussed in Section 6. Section 7 is devoted to discuss experimental results, and comparative remarks with other methods are also provided. Conclusions and suggestions on future work end the paper.