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3D modeling of fracture in brittle isotropic materials using a novel algorithm for the determination of the fracture plane orientation and crack surface area

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A B S T R A C T

A user defined material model for simulation of brittle fracture in isotropic materials was implemented into the explicit FE code Abaqus/Explicit. The model uses a methodology for calculating the orientation of the fracture plane in isotropic materials in 3D space using an analytical approach. Knowing the orientation of the fracture plane, the cross-sectional area of the fracture surface can in a hexagonal element can be calculated. Regularization is then achieved using the size of the fracture plane. The model is first tested on single-element tests and then applied to more complex test cases such as a short bar with chevron notch or micromechanical analyses of composite materials. The numerical predictions correlate well with experimental evidence from the literature.

1. Introduction

Modeling fracture in engineering materials has been the focus of research for many years. One of the major challenges in modeling fracture is the determination of the orientation of the crack in 3D space as the fracture plane is usually not aligned with the element axes. For example, Puck [1] picked up on Hashin's [2] suggestion of applying Mohr's circle to fracture problems in composites by introducing action plane-related fracture criteria based on the stress exposure. There is an infinite number of planes parallel to the fibers which can become the fracture plane and it is therefore necessary to compute different fracture plane orientations and evaluate the stress exposure value [3]. Pinho et al. [4] adopted this heuristic approach and implemented it into a user subroutine for LS-DYNA. As this heuristic search is computationally very expensive, Wiegand et al. [5] proposed the extended golden section search algorithm to calculating the fracture angle in a composite directly. These approaches are feasible for transversely isotropic composite materials in which the fracture planes parallel to the fibers can be described by a single angle. For isotropic materials however, two angles are required to describe the orientation of the fracture plane in 3D space as shown in Fig. 1.

For isotropic materials it is assumed that the maximum principal stress is oriented normally to the fracture plane. In order to find the direction of the maximum principal stress the eigenvalues and eigenvectors of the stress tensor have to be found. This is a time consuming task requiring the application of numerical algorithms for solving systems of equations. Some of these algorithms depend on iterative solutions, which may cause problems if the start values are not chosen in a way ensuring that the solution converges into different eigenvalues. In this paper Cardano's analytical solution ([6]) is used to determine the eigenvalues of the stress tensor, thus allowing the determination of the orientation of the maximum principal stress vector and the orientation of the fracture plane.

A second problem arising during modeling of fracture in engineering materials is associated with strain localization and the resulting mesh sensitivity. In the spirit of Hillerborg [7], Bazant and Oh [8] suggested regularization approaches smearing the fracture

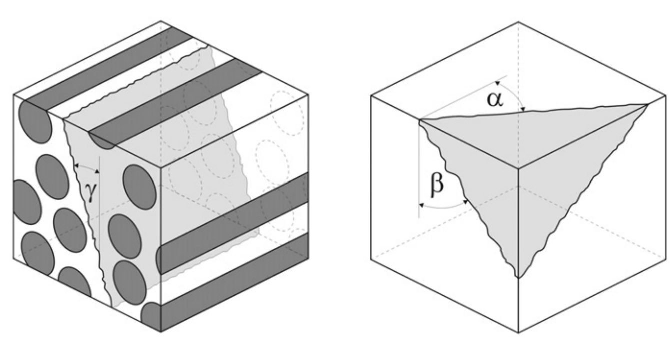

Fig. 1. Description of fracture plane orientations in transversely isotropic materials (left) and isotropic materials (right).

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energy over the volume of the finite element. This approach has since gained in popularity because the smeared finite element formulations have shown to be mesh size independent [4,9,10]. A typical approach of this form is described below:

$$\varepsilon_f = \frac{2 \cdot G_c}{\sigma_0 \cdot L_{char}} \tag{1}$$

where σ_0 is the initiation stress G_c is the fracture toughness, and L_{char} is the characteristic length of the finite element. Pinho [4] suggested to divide the element volume by the cross-sectional area of the crack surface in a finite element in order to calculate the characteristic length:

$$L_{char} = \frac{V_{el}}{A_{crackplane}} \tag{2}$$

In [4], a method is presented for calculating this cross-sectional area for composite materials. For isotropic materials the orientation of the fracture plane can be at any direction in 3D space. This results in a need for a more general method for calculating the area of the crack surface in a finite element. This issue is also addressed in this paper.

The algorithms for determining the orientation of the crack and the cross-sectional area of the fracture surface in a finite element are then included into the framework of an initially elastic material model with damage which can be applied to isotropic materials. The model is implemented as a user material in ABAQUS/Explicit, verified, validated and applied to several test cases.

2. Orientation of the fracture plane

The fracture plane is usually not aligned with the element axes, thus the orientation of the fracture plane needs to be determined in 3D space. This paper deals with intrinsically brittle isotropic materials, therefore we can assume that the maximum principal stress is oriented normally to the fracture plane. In order to determine the direction of the maximum principal stress, the eigenvalues λ and the eigenvectors of the stress tensor have to be calculated. For a one-dimensional stress state the eigenvalues are given below:

$$\lambda_1 = 3 \cdot \frac{2 \cdot (I_1^3/27) - (I_1 \cdot I_2/3) + I_3}{I_2 - (I_1^2/3)} + \frac{I_1}{3} \tag{3}$$

$$\lambda_2 = \lambda_3 = 3 \cdot \frac{2 \cdot (I_1^3/27) - (I_1 \cdot I_2/3) + I_3}{2 \cdot (I_2 - (I_1^2/3))} + \frac{I_1}{3} \tag{4}$$

where I_1, I_2, I_3 are the stress invariants.

For a more general case the eigenvalues can be written as

$$\lambda_1 = \sqrt{-\frac{4}{3} \cdot \left(I_2 - \frac{I_1^2}{3}\right)} \cdot \cos\left(\frac{1}{3} \cdot \arccos\left(-\frac{(2 \cdot I_1^3/27) - (I_1 \cdot I_2/3) + I_3}{2} \cdot \sqrt{-\frac{27}{(I_2 - (I_1^2/3))^3}}\right)\right) + \frac{I_1}{3} \tag{5}$$

$$\lambda_2 = -\sqrt{-\frac{4}{3} \cdot \left(I_2 - \frac{I_1^2}{3}\right)} \cdot \cos\left(\frac{1}{3} \cdot \arccos\left(-\frac{(2 \cdot I_1^3/27) - (I_1 \cdot I_2/3) + I_3}{2} \cdot \sqrt{-\frac{27}{(I_2 - (I_1^2/3))^3}}\right) + \frac{\pi}{3}\right) + \frac{I_1}{3} \tag{6}$$

$$\lambda_3 = -\sqrt{-\frac{4}{3} \cdot \left(I_2 - \frac{I_1^2}{3}\right)} \cdot \cos\left(\frac{1}{3} \cdot \arccos\left(-\frac{(2 \cdot I_1^3/27) - (I_1 \cdot I_2/3) + I_3}{2} \cdot \sqrt{-\frac{27}{(I_2 - (I_1^2/3))^3}}\right) - \frac{\pi}{3}\right) + \frac{I_1}{3} \tag{7}$$

Once the three eigenvalues are known, they are sorted by size using a standard bubble sort algorithm. An iterative von-Mises algorithm is then used to determine the dominant eigenvector, which is normal onto the fracture plane.

Knowing the orientation of the fracture plane in 3D space and based on the assumption that the fracture plane passes through the geometric center of the finite element, one can calculate the crack surface area within each element.

3. Calculation of the cross-sectional area of the fracture plane

In explicit analyses reduced integration elements are used frequently in order to safe computational time. For these elements the single Gauss point is equal to the geometric center of the element. Using the assumption that the geometric center lies within the fracture plane, a local coordinate system $x'y'z'$ is introduced with the origin lying in the geometric center. The fracture plane can then easily be described in Hesse's normal form:

$$\underline{n} \cdot \underline{x} = 0 \tag{8}$$

Knowing the element dimensions the vectors describing the element edges can be computed easily using vector algebra: for a plane fracture plane, the three cases of 4, 6 or 8 intersections of the fracture plane with the element edges have to be considered, see Figs. 1–3. The calculation of intersection points is now explained using the example of the intersection of the fracture plane with the edge P_3P_4 in Fig. 2.

Defining the element coordinate system such that the origin is the geometric center of the element, the element x'-axis is oriented in direction $\underline{P_1P_4}$, the y'-axis is oriented in direction $\underline{P_2P_1}$ and the element z'-axis is oriented in such way that the resulting coordinate system is an orthogonal base as shown in Fig. 2. The corners P_3 and P_4 are then described in local element coordinates: P_3 $(0.5L_1, 0.5L_2, -0.5L_3)$; P4 $(0.5L_1, -0.5L_2, -0.5L_3)$ where L_1, L_2 and L_3 are the edge lengths. Thus, the edge P_3P_4 can be described as follows:

$$\underline{x} = \overrightarrow{OP_3} + \mu \cdot \frac{\overrightarrow{P_3P_4}}{|\overrightarrow{P_3P_4}|} \tag{9}$$

The intersection is then found by equalizing Eqs. (8) and (9) and solving for μ. The fracture plane is intersected by the edge P_3P_4 if $\mu \in [0, L_2]$. Substituting μ into Eq. (9) gives the coordinates of the intersecting point I_6. The other 11 edges have to be checked analogously (Fig. 4).

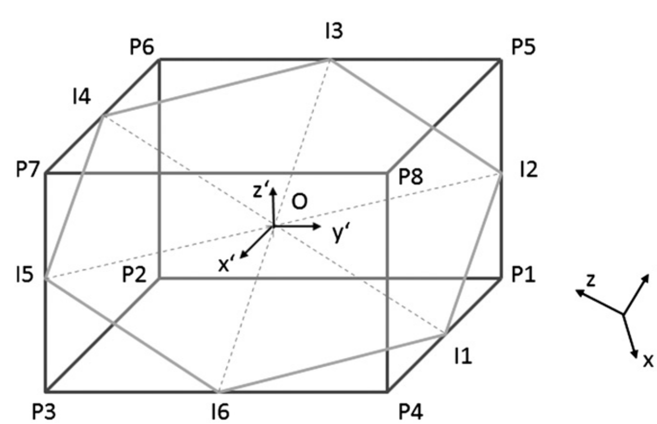

Fig. 2. Exemplary fracture plane in a finite element for the case of 6 intersections.