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Prestress stability of pin-jointed assemblies using ant colony systems

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ABSTRACT

Prestress stability is the key of whether a pin-jointed assembly could be transformed into a tensegrity structure. This study developed an optimization model to investigate the prestress stability of pin-jointed assemblies. The continuous optimization problem was converted into a modified traveling salesman problem (TSP), and the ant colony system (ACS) was used to search for feasible solutions. Coefficients for the independent states of self-stress were taken as different cities in the network. Several typical examples were tested. It could be concluded that the proposed technique is efficient, and applicable to both planar and three-dimensional complex pin-jointed assemblies.

1. Introduction

Tensegrity structures are attracting widespread interest of architects and structural engineers due to their remarkable configurations (noting that the tensegrity structures referred to in this paper include conventional tensegrities, tension structures and most prestressed pin-jointed structures). They are mechanisms while stating in the initial configurations, yet they become stable structures after achieving feasible prestresses. A statically and kinematically indeterminate pin-jointed assembly will be a tensegrity structure, if it could acquire stable equilibrium and possess adequate stiffness. Therefore, prestress stability is the key of whether an indeterminate pin-jointed assembly could be transformed into a tensegrity structure. Identifying the prestress stability is beneficial to better understanding and developing tensegrity structures.

For the basic theory for mechanics of statically and kinematically indeterminate pin-jointed assemblies, the work of Pellegrino and Calladine (1986) could be referred. Two conventional methods to explore the prestress stability of pin-jointed assemblies are the energy method (Connelly, 1982; Motro, 2003; Skelton and Oliveira, 2009), and the equilibrium method (Pellegrino and Calladine, 1986; Quirant et al., 2003; Guest, 2006; Zhang et al., 2009; Tran and Lee, 2010). Calladine and Pellegrino (1991) proposed a linear algorithm based on the equilibrium matrix, to identify whether any states of self-stress would impart first-order stiffness to an inextensional mechanism mode of a pin-jointed assembly. The determination criterion proposed by them has been used as the prestress stability condition. Then Vassart et al. (2000) introduced an analytical method for determining the order of mechanisms of pin-jointed systems, based on the energy method and the variations of member length. The method is helpful to distinguish the finite mechanisms. Sultan et al. (2001) deduced the prestress stability conditions for tensegrity structures using the principle of virtual work, and gave a general expression varied by the tensions of cables. All aforementioned methods convert the prestress stability problems into explicit expressions of specific variables. However, they will become rather inefficient and powerless, if adopted for complex structures with large numbers of nodes and members.

On the contrary, heuristic search methods such as the genetic algorithm (GA), the simulated annealing algorithm (SA), and the ACS do not depend on the initially selected variables. They have powerful global searching ability in large solution spaces. Searching and optimization of initial prestresses for regular tensegrity structures were conducted by the GA (El-Lishani et al., 2005; Xu and Luo, 2010b) and the SA (Xu and Luo, 2010a). The methods introduced in their work optimized the initial prestresses for some specific structures with simple topology and a small number of nodes. Nevertheless, it would be difficult to investigate the assemblies with much more nodes and states of self-stress. To overcome this difficulty, we adopt a different optimization model and an efficient algorithm based on the ACS. Prestress stability of some pin-jointed assemblies which have many nodes, elements, and complex geometries will be investigated in this study.

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2. Prestress stability condition

Consider a pin-jointed assembly in a d-dimensional space (d = 2, 3). The structure consists of n nodes, b members, and g kinematic constraints. Assume that: (a) all the members are straight and have a linear stress–strain relationship; (b) no external loads act on the assembly; and (c) the assembly has m independent modes of inextensional mechanism and s states of self-stress. Then the structure is said to be prestress stable, when the prestress stability condition proposed by Calladine and Pellegrino (1991) is satisfied, given as:

$$\boldsymbol{\beta}^T \sum_{i=1}^{s} (\alpha_i \mathbf{P_i}^T \mathbf{D}) \boldsymbol{\beta} > 0 \tag{1}$$

where $\boldsymbol{\beta}$ is a $m \times 1$ nonzero vector; α_i is the scalar coefficient for the ith state of self-stress; the $(nd - g) \times m$ matrix $\mathbf{P_i}$ contains the product forces of the ith state of self-stress; and the $(nd - g) \times m$ matrix $\mathbf{D}$ represents the modes of inextensional mechanism. It should be noted that $d(d+1)/2$ rigid-body motions have been excluded from the mechanisms' matrix $\mathbf{D}$, if the assembly is free-standing (g = 0). The above inequality could be guaranteed when the $m \times m$ square matrix $\mathbf{Q}$ is positive definite, where

$$\mathbf{Q} = \sum_{i=1}^{s} (\alpha_i \mathbf{P_i}^T \mathbf{D}) \tag{2}$$

For assemblies with more than one independent state of self-stress, many different choices of the coefficient α_i in Eq. (1) are available. Thus, it is difficult using the conventional methods to seek the right coefficients, and to guarantee the matrix $\mathbf{Q}$ to be positive definite. An optimization algorithm is preferred in order to choose a feasible combination of the states of self-stress that stabilize all the mechanisms of the assembly.

Since all the eigenvalues of a positive definite matrix are positive, the number $\bar{q}$ of nonpositive eigenvalues of the matrix $\mathbf{Q}$ is implicitly determined by the coefficients, written as:

$$\bar{q} = f(\boldsymbol{\alpha}) \tag{3}$$

where $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, ..., \alpha_s]^T$, and all the coefficients α_i (i = 1, 2, ..., s) could be bounded in [− 1, 1], as the combined state of self-stress is dimensionless (in fact it is just a normalized form of prestress). $f(\boldsymbol{\alpha})$ is the function representing the number of nonpositive eigenvalues. The matrix $\mathbf{Q}$ is positive definite when $\bar{q} = 0$. Therefore, the optimization problem for determining the prestress stability of a pin-jointed assembly could be formulated as:

$$\begin{array}{ll} \text{minimize} & f(\alpha) \\ \text{subject to:} & \alpha_i \in [-1, 1] \quad i = 1, 2, \ldots, s \end{array} \tag{4}$$

If a feasible solution to $f(\boldsymbol{\alpha}) = 0$ is obtained, the corresponding feasible prestress mode $\mathbf{t}$ will be given by:

$$\mathbf{t} = [t_1, t_2, \ldots, t_b]^T \tag{5}$$

where the normalized mode of prestress $\mathbf{t} = \mathbf{t}(\boldsymbol{\alpha})$ is obtained from the product between the coefficients and the corresponding states of self-stress, and it transforms the pin-jointed assembly into a stable tensegrity structure.

As some members of a tensegrity structure are designed to support tension and others to support compression exclusively, the signs of the prestress mode should be considered in the process of optimization (Yuan and Dong, 2003). Define a $b \times b$ diagonal matrix $\mathbf{T}$ to describe the member types, written as

$$\mathbf{T} = \begin{pmatrix} \mathbf{T}_{11} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \mathbf{T}_{bb} \end{pmatrix} \tag{6}$$

where $\mathbf{T}_{ee} = -1$ means the member e = 1, 2, ..., b is appointed as a compression bar, $\mathbf{T}_{ee} = 1$ represents the member e is a tension cable, and $\mathbf{T}_{ee} = 0$ indicates the member e has not been appointed. Then, the number of negative values $u(\boldsymbol{\alpha})$ in the matrix $\mathbf{T} \cdot \mathbf{t}$ is the actual number of members on which the condition of appointed member types is unsatisfied. $u(\boldsymbol{\alpha})$ is converted to

$$\bar{u}(\boldsymbol{\alpha}) = \frac{u(\boldsymbol{\alpha})}{u(\boldsymbol{\alpha}) + 1} \tag{7}$$

to guarantee that $\bar{u}(\boldsymbol{\alpha}) \in [0, 1)$.

However, from a designer's point of view, the final optimized prestress mode should significantly reduce the complexity of sections of members and the construction cost. Hence, another function $\varepsilon(\boldsymbol{\alpha})$ is defined to describe the unevenness of the prestress modes (Xu and Luo, 2010a), given by

$$\varepsilon(\boldsymbol{\alpha}) = \frac{\sqrt{\frac{\sum_{e=1}^{b} (|\mathbf{t}_e| - \overline{|t|})^2}{b}}}{\overline{|t|}} \tag{8}$$

where $\overline{|t|} = (\sum_{e=1}^{b} |\mathbf{t}_e|)/b$ is average of the absolute value of the prestress modes. Similarly, this new function is transformed into

$$\bar{\varepsilon}(\boldsymbol{\alpha}) = \frac{\varepsilon(\boldsymbol{\alpha})}{\varepsilon(\boldsymbol{\alpha}) + 1} \tag{9}$$

to guarantee that $\bar{\varepsilon}(\boldsymbol{\alpha}) \in [0, 1)$. Then the final objective function is written as

$$\begin{array}{ll} \text{minimize} & Num(\boldsymbol{\alpha}) = [f(\boldsymbol{\alpha}) + C] \times [f_{penalty}(\boldsymbol{\alpha}) + 1]^{\chi} \\ \text{Subject to:} & \alpha_i \in [-1, 1] \quad i = 1, 2, \ldots, s \end{array} \tag{10}$$

where $Num(\boldsymbol{\alpha})$ is the penalized object function; $f_{penalty}(\boldsymbol{\alpha}) = \bar{u}(\boldsymbol{\alpha}) + \bar{\varepsilon}(\boldsymbol{\alpha})$ is the penalty function, to consider the appointed member types (Eqs. (6)–(7)), and to get optimized distribution of prestresses (Eqs. (8)–(9)). A constant positive number C is introduced in Eq. (10) to assure that the object value $Num(\boldsymbol{\alpha})$ is greater than 0, and hence to avoid the penalty function losing weight when $f(\boldsymbol{\alpha}) = 0$. The constant χ is to control the weight of the penalty function.

3. Ant colony systems

The optimization problem described by Eqs. (7), (9), and (10) could be converted into a modified TSP based on the ACS. Therefore, a brief introduction to concepts of the ACS and TSP is also presented in this section.

3.1. General aspects of ACS

The ant colony algorithm was introduced by Dorigo and Gambardella (1997) and Dorigo et al. (2006), to solve some combinatorial optimization problems such as the TSP and the quadratic assignment problem. Since then, it has been successfully adopted to some structural designs and topology optimizations in the engineering field (Camp and Bichon, 2004; Kaveh and Shojaee, 2007; Kaveh and Talatahari, 2010). The ACS is a bionic algorithm inspired by the path searching method of ant colonies in nature. The foraging behavior, which is an important character of an ant colony, leads ants to find the optimal path from foods to their nest, and vice versa. The ant colonies communicate path information through the pheromone trails they deposit. As the consistence of pheromone reflects the experience acquired by the ants, it is proportional to the produced solutions. The shorter the tour made by an ant, the greater the consistence of pheromone it deposits along the paths. Subsequently, the rising pheromone trail is likely to attract much more ants. However, the pheromones will evaporate over time, to avoid the situation in which all ants complete the same tour and obtain local optima.