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W-SPSA in practice: Approximation of weight matrices and calibration of traffic simulation models

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ABSTRACT

The development and calibration of complex traffic models demands parsimonious techniques, because such models often involve hundreds of thousands of unknown parameters. The Weighted Simultaneous Perturbation Stochastic Approximation (W-SPSA) algorithm has been proven more efficient than its predecessor SPSA (Spall, 1998), particularly in situations where the correlation structure of the variables is not homogeneous. This is crucial in traffic simulation models where effectively some variables (e.g. readings from certain sensors) are strongly correlated, both in time and space, with some other variables (e.g. certain OD flows). In situations with reasonably sized traffic networks, the difference is relevant considering computational constraints. However, W-SPSA relies on determining a proper weight matrix (**W**) that represents those correlations, and such a process has been so far an open problem, and only heuristic approaches to obtain it have been considered.

This paper presents W-SPSA in a formally comprehensive way, where effectively SPSA becomes an instance of W-SPSA, and explores alternative approaches for determining the matrix **W**. We demonstrate that, relying on a few simplifications that marginally affect the final solution, we can obtain **W** matrices that considerably outperform SPSA. We analyse the performance of our proposed algorithm in two applications in motorway networks in Singapore and Portugal, using a dynamic traffic assignment model and a microscopic traffic simulator, respectively.

1. Introduction

Due to the well known complexity of transportation systems in our cities, together with their fundamental role in terms of environment, quality of life and economic growth, research in analysis and prediction of traffic phenomena is gaining a growing importance. This has been even more notable with the recent sensing and data processing innovations of varying nature (e.g. telecom, smart cards), globally referred to as "big data". We do have more data, more computing power and higher recognition of the importance of understanding traffic in our cities.

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However, the problem is still very complex as it quickly reaches high dimensionality with large networks, multiple measurements, several traffic control systems, and high and heterogeneous demand patterns. An approach to deal with this complexity is by using simulation models. In this case, the origin–destination (OD) flows (the demand) are assigned to the system by moving vehicles on the network (the supply). This approach can capture emergent behaviour (e.g. congestion) that is often hard to predict analytically. To run properly, simulation models expect, therefore, both supply and demand inputs and parameters. The size and type of such parameter set depends on the simulation scenario and on the simulator itself. One may need to define, for example, OD matrices and route choice model parameters for the demand and speed/density relationship functions or driving behaviour model parameters for the supply.

The essential challenge then becomes the calibration of all the supply and demand parameters in order to reflect the real phenomena. Different requirements are expected for dynamic traffic assignment models (DTA) (e.g. Ben-Akiva et al., 2010a) and for microscopic traffic simulation (e.g. Yang and Koutsopoulos, 1996). For example, DTA models usually utilise mesoscopic demand and supply simulator components, that employ a mix of microscopic and macroscopic models to capture the decision of the travellers and the movement of vehicles throughout the network. They consider the (often thousands or tens of thousands) OD flows in the network as inputs that need to be calibrated. Similarly, in the supply side, segment output capacities are among the parameters that need to be calibrated, and these are easily in the order of thousands. Microscopic traffic simulator models also require OD flows as inputs, but on the supply side they require a much smaller number of parameters to be calibrated (used in the individual models, such as car-following, merging, lane-changing) (Toledo et al., 2007).

Overall, a traffic model may contain hundreds of thousands of parameters, and, in a complex network with a large population, the simulation itself is not computationally negligible. Moreover, due to the (generally unknown) nature of the search space, this becomes a complex optimisation problem. Given the available data (e.g. traffic volumes, densities and speeds from conventional counters, but also travel times or route-choice fractions), the optimisation problem consists of estimating the parameters that minimise the difference between sensed values and simulated values. Of course, the computational costs forbid brute force solutions and the lack of a precise analytical model frustrates the use of deterministic methods. We need a methodology that is parsimonious with the simulation runs, yet capable of making an efficient search in a stochastic fashion.

The Simultaneous Perturbation Stochastic Approximation (SPSA) algorithm (Spall, 1998) was designed to address these issues. Briefly, at each iteration, it generates a pair of new vectors to inspect (i.e. run the simulation for), where each individual vector element, or parameter, is determined by a perturbation with respect to the original value. The particularity is that all parameters are perturbed simultaneously in a stochastic, pair-wise symmetric fashion: the new pair of values of parameter i will be $a_i \pm p_i$, being a_i the original value of parameter i and p_i the perturbation. The gradient is then calculated taking into account the respective simulation results.

The characteristics of SPSA allow for another functionality, introduced by Balakrishna (2006), that is to simultaneous calibrate all supply and demand parameters together, as opposed to have them calibrated separately (and possibly iteratively). Balakrishna (2006) have shown that simultaneous approaches outperform the traditional iterative framework when applied to the calibration of DTA models. SPSA and its variations have since been applied extensively in the field of traffic simulation model calibration. Balakrishna et al. (2007) apply SPSA for the simultaneous calibration of the demand and supply parameters and inputs to the microscopic traffic simulation model MITSIMLab (Yang and Koutsopoulos, 1996) using the network of Lower Westchester County, NY, to demonstrate the feasibility, application, and benefits of the proposed methodology. Ma et al. (2007) compare the performance of SPSA against a genetic algorithm (GA) and a trial-and-error iterative adjustment algorithm (IA) for the calibration of a microscopic simulation model in a northern California network and conclude that SPSA can achieve the same level of accuracy as the other two with a significantly shorter running time. Vaze et al. (2009) present a framework for the joint calibration of demand and supply model parameters of DTA models using multiple sources of traffic information. The calibration problem has been formulated as a stochastic optimisation framework and SPSA was found to outperform competing algorithms, based on results using both counts and travel time measurements obtained from automated vehicle identification systems on a synthetic network and the network of Lower Westchester County, NY.

Huang et al. (2010) applied SPSA for the calibration of dynamic emission models. This research uses a microscopic traffic simulator and the aggregate estimation ARTEMIS as a standard reference. Lee and Ozbay (2008) propose a Bayesian calibration methodology and applied a modified SPSA algorithm to solve the calibration problem of a cell transmission based macroscopic traffic model. In this formulation, the probability distributions of model parameters are considered instead of their point values. Paz et al. (2012) calibrate all the parameters in CORSIM models simultaneously using SPSA and demonstrate its effectiveness.

In the first case study presented in this research, using a mesoscopic DTA model for the entire expressway system in Singapore, it was found that, although SPSA maintained its computational efficiency, its performance in terms of convergence rate and long run accuracy deteriorated significantly, as the problem scale increased. The errors stopped to decrease at relatively high values. Different values of algorithm parameters were tested and adaptive step sizes (a_k) were implemented. However, no significant improvement was made (Lu, 2014). This led us to believe that SPSA itself has fundamental limitations, when applied to very large scale, noisy problems without analytical representation and with correlated parameters and measurements, as identified in Lu et al. (2015), Cipriani et al. (2011), and Cantelmo et al. (2014). One of those limitations refers to the agnostic perspective on the correlation structure between the variables involved and the observations. It is assumed that they are all equally co-dependent, but in practice it is rarely the case.