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Discrete Optimization

Competitive facility location problem with attractiveness adjustment of the follower: A bilevel programming model and its solution

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ABSTRACT

We are concerned with a problem in which a firm or franchise enters a market by locating new facilities where there are existing facilities belonging to a competitor. The firm aims at finding the location and attractiveness of each facility to be opened so as to maximize its profit. The competitor, on the other hand, can react by adjusting the attractiveness of its existing facilities with the objective of maximizing its own profit. The demand is assumed to be aggregated at certain points in the plane and the facilities of the firm can be located at predetermined candidate sites. We employ Huff's gravity-based rule in modeling the behavior of the customers where the fraction of customers at a demand point that visit a certain facility is proportional to the facility attractiveness and inversely proportional to the distance between the facility site and demand point. We formulate a bilevel mixed-integer nonlinear programming model where the firm entering the market is the leader and the competitor is the follower. In order to find the optimal solution of this model, we convert it into an equivalent one-level mixed-integer nonlinear program so that it can be solved by global optimization methods. Apart from reporting computational results obtained on a set of randomly generated instances, we also compute the benefit the leader firm derives from anticipating the competitor's reaction of adjusting the attractiveness levels of its facilities. The results on the test instances indicate that the benefit is 58.33% on the average.

1. Introduction

Competitive facility location (CFL) problems differ from the classical facility location problems considered in the operations research area because they incorporate the competition among the facilities belonging to different firms. The new facility or facilities to be located by a firm have to compete with the facilities of the other firm(s) that are already (or will be) present in the market in order to capture market share. The nature of the competition is the primary factor determining the class of the CFL problem. The two main classes are simultaneous-entry CFL problems and sequential-entry CFL problems. The competing firms simultaneously decide on their facility locations in simultaneous-entry CFL problems, whereas there exists a priority among the competing firms with regard to the timing of the actions in sequential-entry CFL problems. Some of this second class of CFL problems can also be considered as a Stackelberg type of game between two or more competing firms.

Another categorization of CFL problems can be made with respect to the customer choice rules for patronizing facilities: deterministic utility models and random utility models. Although the attractiveness level of a facility is determined by a function of its attributes, and customers are attracted to them according to some utility function in both types of models, there is an important difference between them. In deterministic utility models, the customers visit only the facility which gives the highest utility to them, whereas in random utility models customers visit each facility with a certain probability. This probability can be interpreted as the long-run average proportion of time a customer visits a certain facility. The most widely used random utility model in the CFL literature is the gravity-based model used by Huff (1964, 1966). In this model, the probability that a customer patronizes a facility is proportional to the attractiveness of the facility and inversely proportional to a function of the distance between the customer and the facility. The facility attractiveness is a function of a multitude of attributes related to the facility. For example, when the facility in question is a shopping mall, these factors are mostly various characteristics or attributes of the mall such as the variety of stores, availability of food court/restaurants, adequate parking, accessibility by public transportation, selling price of products and existence of movie theaters. In his seminal work, Hotelling (1929) developed a model which involves two equally attractive ice-cream sellers along a beach strip where customers patronize the closest one. This very first model was extended later for unequally attractive facilities, which is a more realistic assumption given the current situation of today's diversified market.

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In CFL models all competitors in a market can be considered as players of a game, where players act either simultaneously or sequentially to optimize their individual objective functions. As mentioned previously, the players of the game (i.e., the firms locating facilities) decide on their action simultaneously in simultaneous-entry CFL problems, whereas players make their decisions sequentially in sequential-entry or Stackelberg-type CFL problems. We briefly review the literature for the former class of problems first, and then focus on the papers that study Stackelberg-type of CFL problems. We also state the distinction of the problem addressed in this paper from the second class of CFL problems.

1.1. Simultaneous-entry CFL problems

Most of the studies in this class consider the competition of the firms for a single homogeneous product as a two-stage game, where in the first stage both firms simultaneously decide where to locate the facilities. As soon as these decisions are made, they become known to both firms and they continue with the second stage of the game by simultaneously deciding either on the production quantities to supply the markets or on their products' prices. Markets are usually located on the vertices of a network and firms aim at maximizing their profits. Based on some conditions, the uniqueness and existence properties of a Nash equilibrium for quantities or prices are shown. By means of additional conditions, the equilibrium locations are proved to be the vertices of the network. Among the papers including this type of analysis, we can mention Labbé and Hakimi (1991), Lederer and Hurter (1986), Lederer (1986), and Lederer and Thisse (1990). Sarkar et al. (1997) extend the work of Labbé and Hakimi (1991) for multiple firms and nonlinear price functions. Wendell and McKelvey (1981) consider only the locational game for two competing firms on a graph, and seek for a locational equilibrium such that a firm can capture at least 50% of the customers regardless where its competitor is located. By making use of the voting theory, necessary and sufficient conditions for locational equilibrium are developed. Rhim et al. (2003) extend the spatial competition on a discrete space to an oligopolistic three-stage game, where in the first stage facility locations, in the second stage facility capacities, and in the third stage quantities to be produced are decided. Pérez et al. (2004) investigate only the second stage of the game, where the competition between multiple firms takes place only in terms of prices.

1.2. Sequential-entry CFL problems

The CFL models in which players act sequentially can be further divided into two groups. In the first group, the entrant firm aims to open new facilities in a market where there are one or more competitors having existing facilities. However, they do not react to the entrant firm. Such models can be considered as a sequential-entry CFL model since the competitors have moved first and entered the market before the new entrant firm. The latter decides on its strategy about opening new facilities being aware of the location and attractiveness levels of the existing facilities. The following papers address problems in this group: Achabal et al. (1982), Berman and Krass (2002), Drezner et al. (2002), Benati and Hansen (2002), Drezner and Drezner (2004, 2006, 2008), Fernández et al. (2004), Aboolian et al. (2007a,b), Fernández et al. (2007) and Tóth et al. (2009).

In the second group of CFL models, the competitor having existing facilities reacts to the new firm subsequent to its market entry. This situation leads to the so-called two-level or bilevel optimization problem in which there are two independent players called leader and follower. These players act in a sequential manner with the aim of optimizing their own objective functions, which are almost always in conflict with each other as pointed out by Moore and Bard (1990). In this setting, the leader first makes a decision (or selects a strategy) to optimize its objective function with the foresight or anticipation that given this decision the follower will optimize its own objective function. A bilevel programming (BP) formulation in optimization corresponds to a Stackelberg game in the context of game theory (Bard, 1998).

When we review the literature by focusing on the game-theoretic formulations within the context of CFL problems, we come across to the following 10 studies. Some of them develop BP models, while others solve Stackelberg equilibrium problems. The book by Miller et al. (1996) focuses on the equilibrium facility location modeling. The developed mathematical models therein consider a firm which has to simultaneously decide on the production, and distribution of a single, homogeneous product such that there exists an equilibrium in the market represented as a network. The entering firm, usually called a Stackelberg or leader firm, anticipates the reaction of the follower firms that have existing facilities in the market. These followers are assumed to be Cournot firms trying to achieve a Nash equilibrium by making changes in their production and distribution levels as a reaction to the leader firm. However, these firms operate under the Cournot assumption that the others do not change their production and distribution levels. The novelty here is that the Cournot–Nash equilibrium of the followers is represented as a variational inequality formulation. In order to compute the Cournot–Nash and Stackelberg–Cournot–Nash equilibria, many heuristic algorithms employing sensitivity analysis are suggested.

Fischer (2002) considers a discrete CFL with two competitors. Each competitor sells the same product to customers which are aggregated at discrete points in space called markets. One of the competitors becomes the leader and the other takes the role of the follower. Both of the decision makers want to determine both the locations of a fixed number of new facilities to be established from a set of potential sites and the price of the product at each market. It is assumed that the product can be sold at different prices at different markets (i.e., discriminatory pricing) where the price at a market depends on the distance from the facility serving the market. Customers prefer to make the purchase from the competitor offering the lowest price. Fischer (2002) formulates two bilevel models: a mixed-integer nonlinear bilevel model in which both players fix their locations and prices once and for all, and a linear bilevel model with binary variables where price adjustment are possible. A heuristic solution procedure is developed to solve the linear bilevel model, but no computational results are provided.

Bhadury et al. (2003) suggest a centroid model in the continuous space, where the follower locates extra facilities as a reaction to the leader's action. Drezner (1982) introduces two problems. In the first one, a new facility is located so as to attract most of the buying power from the demand points when there is already an existing facility in the market, whereas the second problem involves again the location of a new facility with the same goal, but this time by taking into account the possibility that the competitor opens a facility in the future. Plastria and Vanhaverbeke (2008) consider the maximal covering model and incorporate into this model the information that a competitor will enter the market with a new facility in the future. The objective of the leader is to locate facilities under a budget constraint in order to maximize the market share after the competitor's entry. The authors formulate three models corresponding to three strategies: worst-case analysis (maximin strategy), the minimum regret strategy, and the Stackelberg strategy which corresponds to taking into account the objective function of the competitor. Serra and ReVelle (1993) develop a model where both the leader and follower locate an equal number of facilities which are visited by customers only if they are the closest ones. In this model, the objective of the leader is to minimize the market share of the follower, and two heuristic algorithms are proposed for its solution.