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A bi-level programming model for protection of hierarchical facilities under imminent attacks

Nasrin Aliakbarian, Farzad Dehghanian*, Majid Salari

Department of Industrial Engineering, Faculty of Engineering, Ferdowsi University of Mashhad, Mashhad, Iran

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ABSTRACT

Disorders caused by deliberate sabotage and terrorist attacks have always been considered as a major threat by the governments. Hence, identifying and planning for strengthening of critical facilities have become a priority for more security and safety. This paper presents a bi-level formulation of the *r*-interdiction median problem with fortification for critical hierarchical facilities. In the developed bi-level formulation, the defender, as the leader, decides to protect a certain number of facilities in each level of the hierarchical system in order to minimize the impact of the most disruptive attacks to unprotected facilities. On the other hand the attacker, as the follower, with full information about protected facilities, makes his interdiction plan to maximize the total post-attack cost incurred to the defender. We develop three metaheuristic algorithms and an exhaustive enumeration method to solve the introduced problem. Extensive computational tests on a set of randomly generated instances demonstrate the effectiveness of the developed algorithms.

1. Introduction

There are a number of examples showing that the terrorists primarily aim at creating chaos through disrupting services or delaying in emergency response systems rather than killing the people. The attack to telecommunication towers in Afghanistan occurred by Taliban [1], the assault on an ambulance station in Northern Ireland [2] and the possibility of disruptive threats to the electric grids [3] are examples of such attacks. So, a major concern in recent decades is to guarantee stability and efficiency in service provision against a sudden loss of facilities' capacity due to the occurrence of deliberate attacks.

Responding to these threats, the governments have studied to identify the critical system components and to promote their safety against such attacks. In particular, critical components consist of those certain physical assets that, when lost, result in significant disruption or disorder in operational capabilities of the system [4]. The list of key assets and critical infrastructures includes systems such as telecommunication, electrical power, banking and finance. Moreover, it also contains facilities for providing emergency services or locations to store gas and oil products [5]. In addition, protection of these facilities can be enhanced by hardening facility perimeters, limiting access, moving critical facility locations to interior and safe areas and developing backup power systems [6].

There are some critical facilities and infrastructures that have a hierarchical structure in nature. Health care delivery system, oil wells and refineries, arms stockpiles and food warehouses are some examples of such systems in which facilities are consisting of several levels. In such a system, facilities consist of k levels where the lowest level is called level 1 and the highest level is called level k. Hierarchical system of facilities is classified as *single-flow* or *multi-flow* based on its flow pattern. In a *single-flow* system, the flow of customers and/or goods starts from the lowest level, passes through all levels, and finally ends at the highest level, and vice versa. In contrast, in a *multi-flow* system, the flow can be from any lower (higher) level to any higher (lower) level. From another point of view, a hierarchical system can be classified as *nested* or *non-nested* according to the service availability at the levels of the hierarchy. In a *nested* hierarchy, a higher-level facility provides all services supplied by a lower-level facility and at least one additional service while in a *non-nested* hierarchy, facilities on each level offer different services [7].

In this paper, we aim at developing the traditional r-interdiction median problem with fortification (RIMF) [8] for multi-flow nested hierarchical facilities. Essentially, the goal is to identify the most cost-effective way of protecting facilities in each level of the hierarchy against worst-case disruptions.

* Correspondence to: Department of Industrial Engineering, Faculty of Engineering, Ferdowsi University of Mashhad, PO Box 91775-1111, Mashhad, Iran.
Tel.: +98 51 38805110; fax: +98 51 38763301.
E-mail addresses: Nasrin.aliakbarian@yahoo.com (N. Aliakbarian), f.dehghanian@um.ac.ir (F. Dehghanian), msalari@um.ac.ir (M. Salari).

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The rest of the paper is organized as follows. In the next section we review the relevant literature on the identification and protection of critical infrastructures. Section 3 presents the model and its properties. The details of the solution methods are given in Section 4. Section 5 includes the computational study on a set of randomly generated test instances. In Section 6 we provide a decision rule for identifying critical facilities for protection. Finally, Section 7 concludes the paper with suggestions for future researches.

2. Literature survey

Generally, various proposed interdiction models in the literature are classified into two categories, namely "network interdiction models" and "facility interdiction models". In the first category, there are many papers devoted to the design of survivable networks or protection of existing ones. In the majority of these researches, attackers are looking for the elimination of a subset of network arcs that leads to the maximum reduction in the network capacity or maximum increment in the shortest path between two specified origin and destination vertices. Wollmer [9] was among the first authors who studied the arc interdiction model and presented an algorithm that minimizes the maximum amount of flow between facilities by removing a certain number of arcs. Given a limited interdiction budget, McMasters and Mustin [10] presented a mathematical model to determine the optimal interdiction plan for minimizing the maximal flow capacity from a source to a sink vertex of a given network. In their model, it is assumed that each arc can be interdicted partially. Numerous researchers have considered the network interdiction problem from which we can refer to the papers by Wood [11], Ghare et al. [12], Cormican et al. [13], Israeli and Wood [14] and Lim and Smith [15].

The facility interdiction models involve the problems that emphasize the destruction of the most vital facilities in a given network. The first published paper in this area is that by Church et al. [16]. They introduced two new models called the *r-interdiction median problem* and the *r-interdiction covering problem*. Both models can be used to identify a subset of r (out of p) existing facilities in a supply system that when eliminated by the attacker, causes the greatest loss of efficiency or coverage of the system. In a subsequent work, Church and Scaparra [4] incorporated the protection of facilities into the r-interdiction median problem to minimize the disruptive effects of possible intentional attacks to the system. In their problem, denoted as IMF, the leader seeks to allocate the protection resources to a subset of existing facilities by examining all possible patterns of losing r facilities; therefore the size of the problem grows rapidly as the number of attacks and the number of facilities increase. In this regard, the method presented in their paper applies only to the problems with the very modest size. Accordingly, in another work, Scaparra and Church [6] reformulated the IMF problem as a maximal covering problem. They provided a new solution approach to reduce the original model size to solve larger size instances of the IMF problem. Although the reduced model can be solved to optimality faster than the former one, it still needs to investigate all possible ways of interdicting r of the p available facilities.

In practice, the protection and interdiction decisions are given by two different decision-makers, i.e. the leader and the follower. This makes the problem as a leader-follower game which is also known as the static Stackelberg game [17]. This offers a relatively new research venue. Scaparra and Church [8] are the first ones who proposed a bi-level programming formulation of the r-interdiction median problem with fortification (RIMF). In the developed bi-level formulation, defender as the leader decides about the facilities to protect in order to minimize the worst-case efficiency reduction due to the loss of the unprotected facilities. Moreover, attacker as the follower tries to incur worst-case scenario losses in the lower-level interdiction model. Aksen et al. [18] considered the budget constraint instead of a predetermined number of facilities to be fortified in the RIMF model. Losada et al. [19] addressed the facility recovery time in a planning horizon framework in cases of probable and improbable disruptions. Liberatore et al. [20] presented the stochastic version of RIMF in which the extent of terrorist attacks and malicious actions are uncertain. Keçici et al. [21] developed another formulation where the leader aims to locate new facilities, relocate existing ones if essential, and protect some of them by considering the budget constraint to ensure a maximum coverage of the customer zones. Moreover, in this paper the gradual or partial coverage concept is considered for the provided services by a network of facilities. Liberatore et al. [22] proposed a tri-level formulation for a facility protection model that considers the propagation of the disruptions from one element of the system to another. In Aksen and Aras [23] the defender decides about facilities to be opened and protected conjunctly. Aksen et al. [24] integrated the partial facility interdiction into the interdiction problem with capacitated facilities and outsourcing option.

Apparently, the common assumption in all of the previous discussed models in the literature, is the similarity of the services provided by different facilities. With this assumption, all of the facilities, regardless of the type of demand allocated to them, have the same importance to the leader and the follower. Furthermore, there is no relationship between facilities for service providing. In a hierarchical system, the attack and defense costs, type of the provided services and importance of the protecting facilities differ at each level; and demands can be satisfied by moving through different facilities located at different levels. Consequently the previous models are not eligible for such systems. For a recent literature survey of hierarchical facility location and its applications, we refer the reader to Farahani et al. [25].

In this paper, we provide a bi-level formulation to protect the critical facilities in a nested multi-flow hierarchical system. In the proposed bi-level formulation, the defender, as the leader, seeks to protect a certain number of facilities in each level with the intention of minimizing the impact of the most disruptive attack to unprotected facilities. In contrast, the attacker, as the follower, destroys a predetermined number of unprotected facilities in each level of the hierarchy to maximize the total cost imposed to the system. The contributions of this paper are twofold: (1) we extend the traditional RIMF model to include hierarchical facilities, and (2) we provide mathematical formulation and several ad-hoc heuristic algorithms to solve the introduced problem.

3. Problem definition and modeling

This problem is inspired from emergency medical services in which some people need clinical services and some others need to be referred to hospitals. In such situation people are first moved to the clinics and after receiving clinical services they are referred to a hospital for further remedy. A similar problem was studied by Galvão et al. [26] in which prenatal health care system had been considered. Essentially, they have considered basic units (level 1), maternity homes (level 2), neonatal clinics (level 3) and general hospitals (level 4) as nested hierarchical facilities. In this system, mothers with low, medium and high risk categories need different service levels from facilities in the appropriate level. They also assume that a given proportion of mothers at maternity homes have to be referred to a neonatal clinic who can be transported there by ambulance.

Consider a supply system with hierarchical nested facilities composed of two levels. We assume that facilities located at the