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Innovative Applications of O.R.

Linear programming models based on Omega ratio for the Enhanced Index Tracking Problem

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ABSTRACT

Modern performance measures differ from the classical ones since they assess the performance against a benchmark and usually account for asymmetry in return distributions. The Omega ratio is one of these measures. Until recently, limited research has addressed the optimization of the Omega ratio since it has been thought to be computationally intractable. The Enhanced Index Tracking Problem (EITP) is the problem of selecting a portfolio of securities able to outperform a market index while bearing a limited additional risk. In this paper, we propose two novel mathematical formulations for the EITP based on the Omega ratio. The first formulation applies a standard definition of the Omega ratio where it is computed with respect to a given value, whereas the second formulation considers the Omega ratio with respect to a random target. We show how each formulation, nonlinear in nature, can be transformed into a Linear Programming model. We further extend the models to include real features, such as a cardinality constraint and buy-in thresholds on the investments, obtaining Mixed Integer Linear Programming problems. Computational results conducted on a large set of benchmark instances show that the portfolios selected by the model assuming a standard definition of the Omega ratio are consistently outperformed, in terms of out-of-sample performance, by those obtained solving the model that considers a random target. Furthermore, in most of the instances the portfolios optimized with the latter model mimic very closely the behavior of the benchmark over the out-of-sample period, while yielding, sometimes, significantly larger returns.

1. Introduction

A financial service company, usually an investment bank, deals with fund management when it directly manages the asset investments on behalf of its customers. Fund management typically includes activities as asset screening and selection, asset trading, monitoring, reporting to stakeholders, and internal audit. In modern financial stock exchanges, market indices have become standard benchmarks for evaluating the performance of a fund manager. Over the last years, the number of funds managed by index-based investment strategies has increased tremendously in different economies such as USA (see Jorion (2002)), Japan (see Koshizuka, Konno, & Yamamoto (2009)), and Australia (see Frino, Gallagher, & Oetomo (2005)). Traditionally, index-based fund management strategies have been broadly categorized into passive and active management.

- A fund manager that implements a *passive management* strategy aims at replicating, as close as possible, the movements of an index of a specific financial market (the so-called *benchmark*), like the S&P 500 in the New York Stock Exchange or the FTSE 100 in the London Stock Exchange. This strategy is usually refereed to as *index tracking*, and aims at minimizing a function (the tracking error) that measures how closely the portfolio tracks the market index to which it is benchmarked. If the manager builds a portfolio containing all the securities constituting the benchmark in the exact same proportions, it is said to follow a full replication strategy. Despite full replication can be seen as the most natural way to track a benchmark, such a strategy is rarely applied in practice mainly due to the impact of transaction costs. Indeed, several researchers point out that transaction and administration costs are typically an increasing function of the number of assets in a portfolio (e.g., see Coleman, Li, & Henniger (2006)). Hence, a better strategy consists in trying to mimic a market index by choosing a subset of the securities constituting the benchmark. In this case, the fund manager is said to follow a partial replication strategy.
- A fund manager that implements an *active management* strategy makes specific investments with the goal of outperforming the

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benchmark. Based on his/her beliefs, the fund manager builds a portfolio where some securities are overweighted and some other underweighted compared to the benchmark trying to exploit possible market inefficiencies (e.g., see Li, Sun, & Bao (2011)). Active management strategies often involve frequent trading to rebalance the portfolio composition in an attempt to beat the benchmark (cf. Lejeune & Samatlı-Paç (2013)), thus generating high transaction costs which diminish the fund return.

Several studies have highlighted pros and cons for each of the two strategies. For instance, there are evidences that a remarkable number of actively managed funds do not outperform its benchmark over the long term (see, among others, Gruber (1996)). As a consequence, fund managers often prefer to follow hybrid strategies using both passive and active fund management to allocate the available capital, usually running a passive strategy to manage the large portion of the fund investment, and employing active strategies to manage only a limited portion of the investment (see Scowcroft & Sefton (2003)).

Enhanced index tracking, sometimes called enhanced indexation, has evolved as a synthesis between the two strategies of passive and active management trying to catch the strengths of both approaches. Indeed, enhanced index tracking is designed to outperform a given benchmark, therefore resembling active management, incurring only a limited additional risk with respect to the benchmark, and thus being similar to passive management. The *Enhanced Index Tracking Problem* (EITP) is the problem of selecting a portfolio of assets able to outperform a market index while incurring a limited additional risk. Nowadays, both researchers and fund managers use optimization models and algorithms to face both the problem of selecting a portfolio that replicates or tries to beat a market index. However, while several mathematical formulations can be found for the index tracking problem, the EITP has been only recently introduced in the literature, and the number of papers addressing this problem is still quite limited.

In this paper, we deal with the EITP using for the first time the Omega ratio, introduced by Keating and Shadwick (2002), as performance measure. The Omega ratio is a performance measure that has two interesting features that make it suitable for the EITP. It assesses the performance against a benchmark, on one side, and it accounts for the asymmetry in returns distributions by separately considering upside and downside deviations, on the other side. Recently, the Omega ratio has become known in portfolio selection where the optimization is considered with respect to a target (see, for instance, Passow (2005)).

Contributions. Given the growing popularity of the Omega ratio, the definition and evaluation of optimization models maximizing this performance measure is a research area that is receiving increasing attention both from academics and practitioners. To the best of our knowledge, this paper is the first attempt to apply the Omega ratio in the context of enhanced indexation. We introduce two optimization models. In the first formulation we apply the basic definition of the Omega ratio where the benchmark is a known value. The second formulation improves on the first one by considering a random target rather than a given benchmark value. We show that both formulations, that are nonlinear in their natural form, can be transformed into Linear Programming (LP) models. We extend both optimization models to include two important real features, that is a cardinality constraint on the number of assets in the portfolio and buy-in thresholds on the weight of each selected stock. The inclusion of real features requires the introduction of additional binary variables, thus transforming the LP formulations into Mixed Integer Linear Programming (MILP) models. The performance of the optimal portfolios selected by the proposed models has been validated through extensive computational experiments carried out on benchmark instances taken from the literature. The computational results show that the portfolios selected by the first formulation that adopts the basic definition of the Omega ratio are consistently outperformed, in terms of out-of-sample performance, by those obtained solving the second formulation that considers a random target. Finally, in most of the instances the portfolios optimized with the latter optimization model track very closely the behavior of the benchmark over the out-of-sample period, while yielding, sometimes, significantly larger returns.

Structure of the paper. The remainder of the paper is organized as follows. In Section 2 we survey the most recent literature on the EITP, and briefly discuss the use of the Omega ratio in optimization problems. In Section 3 we introduce the two mathematical formulations for the EITP based on the Omega ratio. We show how each formulation can be transformed into an LP model, and how to extend the latter models in order to include the described real features. In Section 4 we report on the computational experiments and provide an extensive evaluation of the out-of-sample performance of the optimal portfolios. Finally, in Section 5 some concluding remarks and future research directions are drawn.

2. Literature review

Fund managers are increasingly using optimization models to build their portfolios (see Wilding (2003)). Despite the fact that in the literature several authors tackle and propose mathematical formulations for the index tracking problem (see Beasley, Meade, and Chang (2003), Canakgoz and Beasley (2009), Guastaroba and Speranza (2012) and references therein), the study of the EITP is a relatively recent and growing research area. Indeed, to the best of our knowledge, the first formalization of the EITP is due to Beasley et al. (2003). Canakgoz and Beasley (2009) provided an extensive review of the literature on the enhanced index tracking problem where almost all the papers cited date from 2005 or later. Several papers on the enhanced index tracking are discussed in Guastaroba and Speranza (2012) and in the articles mentioned above. For this reason, we have decided to concentrate the literature review on the papers on the EITP not discussed in those articles.

In the second part of this section, we describe the Omega ratio and briefly review the main related literature.

2.1. Recent literature on the enhanced index tracking problem

A convex minimization model with linear objective function and quadratic constraints for the EITP is proposed by Koshizuka et al. (2009). The authors aim at minimizing the tracking error from an index-plus-alpha portfolio, choosing among the portfolios with a composition highly correlated with the benchmark. The term index-plus-alpha portfolio is sometimes encountered in the literature on enhanced indexation and refers to a portfolio that outperforms the benchmark by a given, typically small, amount α. Two alternative measures of the tracking error are considered in Koshizuka et al. (2009): one based on the absolute deviation between the portfolio and the index-plus-alpha portfolio values, and the other using the downside absolute deviation between these two quantities. Mezali and Beasley (2013) apply quantile regression to index tracking and enhanced indexation. Their model includes, among other characteristics, a cardinality constraint and buy-in thresholds on asset weights. The resulting formulation is a MILP model. Valle, Meade, and Beasley (2014) study the problem of determining an absolute return portfolio and propose a three-stage solution approach. The authors discuss how their approach can be extended to solve the EITP. In Lejeune (2012) the EITP is solved using a game theoretical approach. The problem is formulated as a stochastic model which aims at maximizing the probabilistic excess return of the portfolio compared to the benchmark while ensuring that the relative risk, given by the downside absolute deviation, does not exceed a chosen maximum level.