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An SDP approach for multiperiod mixed 0–1 linear programming models with stochastic dominance constraints for risk management ☆

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ABSTRACT

In this paper we consider multiperiod mixed 0–1 linear programming models under uncertainty. We propose a risk averse strategy using stochastic dominance constraints (SDC) induced by mixed-integer linear recourse as the risk measure. The SDC strategy extends the existing literature to the multistage case and includes both the first-order and second-order constraints. We propose a stochastic dynamic programming (SDP) solution approach, where one has to overcome the negative impact of the cross-scenario constraints on the decomposability of the model. In our computational experience we compare our SDP approach against a commercial optimization package, in terms of solution accuracy and elapsed time. We use supply chain planning instances, where procurement, production, inventory, and distribution decisions need to be made under demand uncertainty. We confirm the hardness of the testbed, where the benchmark cannot find a feasible solution for half of the test instances while we always find one, and show the appealing tradeoff of SDP, in terms of solution accuracy and elapsed time, when solving medium-to-large instances.

1. Introduction

Let $\mathcal{T}$ be a time horizon of length T. Let a_t and c_t be row vectors, b_t a column vector, and $A^t_{t'}$ and $B^t_{t'}$ matrices, all of adequate dimensions. Consider the following multiperiod mixed 0–1 linear programming model:

$$\begin{aligned} \text{minimize} \quad & \sum_{t \in \mathcal{T}} (a_t x_t + c_t y_t) \\ \text{subject to} \quad & \sum_{t'=1}^{t} (A^t_{t'} x_{t'} + B^t_{t'} y_{t'}) = b_t \quad \forall t \in \mathcal{T} \\ & x_t \in \{0,1\}^{nx(t)}, \quad y_t \in \mathbb{R}^{ny(t)} \quad \forall t \in \mathcal{T}, \end{aligned} \tag{MILP}$$

where x_t (resp. y_t) is a vector of 0–1 (resp. continuous) variables. In this paper we consider the uncertainty on the parameters in (MILP) and assume it to be represented by a multiperiod scenario tree, see Fig. 1.

In uncertain environments, it is common to optimize the expected value of the objective function, yielding a risk neutral solution. Because of Risk Management considerations, one would like to implement a risk averse solution to ensure that the variability in the objective function values is low and, in particular, for those on the right tail (for a minimization formulation). A risk measure is called coherent [6] if it satisfies the following four axioms: translation invariance, subadditivity, positive homogeneity and monotonicity. Examples of risk averse measures are scenario optimization [12,17], semi-deviations [1,27], excess probabilities [33], conditional Value-at-Risk [1,29,31] and stochastic dominance constraints strategies [22,23]. We refer the reader to [3] for a state-of-the-art survey on risk averse strategies.

In this paper we use stochastic dominance constraints (SDC) as the risk measure [37]. SDC is a relatively new risk averse strategy [13–15], but very appealing for Risk Management [3,10]. In their typical form, SDC strategies aim to reduce the chance of having scenarios with large objective function values (first-order SDC) or the scenario objective function values themselves (second-order SDC). In addition to the increase in the number of constraints and decision variables, including 0–1, the SDC strategies require cross-scenario constraints. These affect the nice block structure of risk neutral Deterministic Equivalent Models [38], posing a challenge to decomposition approaches, the only viable approaches to deal with large stochastic models [7,9,16,18,36]. Therefore, works on algorithms for SDC strategies do not abound in the literature.

Our contribution is to propose an SDC strategy for the mixed 0–1 linear programming problem (MILP) and a decomposition approach that can successfully deal with cross-scenario constraints. Our SDC

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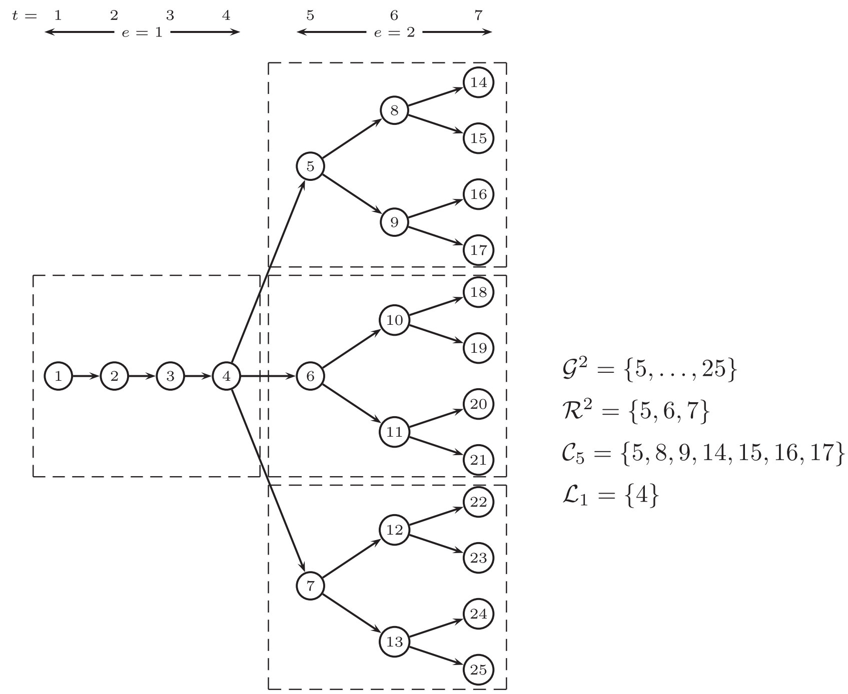

Fig. 1. Breaking the time horizon into stages.

strategy is the multistage extension of the mixture of the first-order SDC in [23] and the second-order SDC in [22] induced by mixed-integer linear recourse for two-stage programs. Thus, in addition to the computational burden coming from SDC, we have multiple stages whose scenario groups need to satisfy the non-anticipative principle. In [19], the multistage SDC model is treated via an exact branch-and-fix coordination methodology that requires an affordable elapsed time but it is only intended for medium-scale instances where linking decision variables appear in a given period and the next one. To be able to handle larger instances and/or linking across more than one period, we propose in this paper a metaheuristic approach.

We extend the stochastic dynamic programming (SDP) metaheuristic presented in [11] to deal with cross-scenario constraints. This metaheuristic approach decomposes the stochastic problem into a collection of subproblems. To solve the subproblems efficiently, piecewise linear convex estimations for the impact of the decisions to be made at a given subproblem on the objective function value related to the future are constructed. The SDP metaheuristic in [11] was designed to solve risk neutral models, and thus in the absence of cross-scenario constraints, where the linking between subproblems is across one period [11,20]. The extension we propose in this paper deals with these two issues using a mechanism that distributes the SDC bounds among the immediate successors of any given subproblem.

We have chosen Supply Chain Management (SCM) as our area of application, where risk neutral models abound in the literature, see e.g. [4,5,34,39] for references in the last decade. Only recently, there have been some attempts to develop solution methods for risk averse SCM models [2,26]. We consider a supply chain planning model, which involves multiple products, multiple periods and a network of players who form the supply chain (typically, markets, production plants, distribution centers and retailers). Raw materials are available at the top of the chain, customers are at the bottom and face demand on a set of end products. The Bill of Materials describes how the players can produce the end products. The goal of the supply chain planning model is to satisfy customer demand at the lowest total costs, where procurement, production, inventory and distribution decisions need to be made.

In the computational experience we benchmark our solution approach against the commercial Mixed Integer Programming (MIP) solver CPLEX [8] in terms of solution accuracy and computation time. We use 12 instances of the SCM model sketched above. These are instances with both 0–1 and continuous variables, where demand is uncertain, and $S2$ sets of variables model nonlinear procurement cost functions, while the SDC risk modelling requires additional 0–1 variables. We illustrate the hardness of the instances, and therefore their adequacy as testbed, in terms of area of application and data generation. From the numerical results, we conclude that the SDP approach is attractive when solving medium-to-large instances, while CPLEX cannot guarantee a feasible solution in half of the instances tested.

The remainder of the paper is organized as follows. In Section 2 we introduce the risk averse SDC formulation. In Section 3 we present the stochastic dynamic programming approach for SDC. We devote Section 4 to the computational experience. Finally, we conclude the paper and discuss future research directions in Section 5.

2. The risk averse SDC strategy

In this section we propose an SDC risk averse strategy for (MILP). This is the multistage extension of the mixture of the first-order SDC in [23] and the second-order SDC in [22] induced by mixed-integer linear recourse for two-stage programs. To avoid scenarios with high objective function values (hereafter, costs), the SDC strategy uses the so-called cost thresholds. For each cost threshold the aim is to have all scenario costs below the threshold. However, this may be too conservative and therefore one allows an excess on the cost threshold. In order to control this excess several mechanisms are used. First, a maximum is set on the size of the excess. Second, an upper bound is imposed on the chance that excess occurs. Third, an upper bound is imposed on the expected excess. Expected excess of scenario costs on the imposed cost thresholds has its roots in Integrated Chance Constraints, which were introduced in [24] and further investigated in [25].

The uncertainty on the parameters is represented by a multiperiod scenario tree, see Fig. 1. There is a one-to-one correspondence