Electrical Power and Energy Systems 62 (2014) 265–272

Contents lists available at ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

A mixed-integer quadratically-constrained programming model for the distribution system expansion planning

John F. Franco *, Marcos J. Rider, Rubén Romero

Faculty of Engineering of Ilha Solteira, Paulista State University, Ilha Solteira, SP, Brazil

ARTICLE INFO

Article history:
Received 29 May 2013
Received in revised form 4 April 2014
Accepted 21 April 2014
Available online 22 May 2014

Keywords:
Distribution system expansion planning
Distribution system optimization
Mixed-integer quadratically-constrained programming

ABSTRACT

This paper presents a mixed-integer quadratically-constrained programming (MIQCP) model to solve the distribution system expansion planning (DSEP) problem. The DSEP model considers the construction/reinforcement of substations, the construction/reconductoring of circuits, the allocation of fixed capacitors banks and the radial topology modification. As the DSEP problem is a very complex mixed-integer non-linear programming problem, it is convenient to reformulate it like a MIQCP problem; it is demonstrated that the proposed formulation represents the steady-state operation of a radial distribution system. The proposed MIQCP model is a convex formulation, which allows to find the optimal solution using optimization solvers. Test systems of 23 and 54 nodes and one real distribution system of 136 nodes were used to show the efficiency of the proposed model in comparison with other DSEP models available in the specialized literature.

1. Introduction

The distribution system expansion planning (DSEP) aims to find the best proposal of expansion for the system that minimizes the investment and operation costs, while satisfying operational constraints, such as voltage magnitude limits at buses, current flow magnitude limits in circuits, maximum apparent power in substations and a radial configuration of the system, for a planning horizon in which demand is known. Thus, the DSEP can add the following to system: (a) new substations and/or repower existing ones; (b) change the conductors of existing circuits and/or build new circuits in branch candidates; and (c) allocate of capacitors and/or voltage regulators, [1].

According to the planning horizon, the DSEP problem can be classified as a short-range (1 to 4 years) or long-range (5 to 20 years) problem [2]. According to the model, one can have a static problem, in which only one stage (planning horizon) is considered, or a dynamic problem (multistage problem), in which several planning horizons are considered in the same problem [3]. The latter problem is not addressed in the present paper. Mathematically, the DSEP problem is modeled as a mixed-integer non-linear programming (MINLP) problem and has been solved using heuristic algorithms, meta-heuristic techniques and also classical optimization techniques such as mixed-integer linear programming (MILP), MINLP and quadratic programming (QP) [4].

Reference [5] presents a MILP model for the integral planning of primary-secondary distribution systems. In [6], a MILP model for the multi-stage DSEP problem is presented considering the available capacities of distributed generators in the system. Mixed-integer programming models determine the optimal location of distribution substation and feeder expansion are developed in [7,8].

In [9,10], QP is used to model the DSEP problem. A heuristic constructive algorithm for the DSEP problem, that approximates the real power losses using a square function is proposed in [9]. The algorithm relaxes the integrality of the decision variables and solves the resultant QP problem to determine the variables that can be rounded. In [10], a two-phases iterative technique is used, that determines the optimal substation sites in the first phase, while the second phase selects the configuration of the network; the integrality of the variables is relaxed allowing to solve a QP problem, and, along with its solution, integer constraints are imposed using heuristic techniques.

In [11], a constructive heuristic algorithm (CHA) is presented to solve the DSEP problem. In this work, the CHA uses a sensitivity index obtained by the solution of a non-linear programming model. Additionally, a local improvement phase is implemented. [12] presents a conic programming model for the DSEP problem. In that paper, two formulations are analyzed: the single-circuit and the parallel equivalent circuit. Additionally, constraints are proposed to eliminate loops with the aim of obtaining a tight formulation

* Corresponding author. Tel.: +55 (18)37431000.
E-mail address: jffranco@gmail.com (J.F. Franco).

http://dx.doi.org/10.1016/j.ijepes.2014.04.048
0142-0615/© 2014 Elsevier Ltd. All rights reserved.

and reducing the computational effort necessary to solve the DSEP problem. A MINLP model for the DSEP problem is proposed in [13], formulating generalized radiality constraints considering multiple substations, distributed generator and transfer nodes (nodes without load). The presented model is solved using an algorithm based on the non-linear branch and bound technique, and the non-linear programming problems are solved using a commercial solver. Heuristic algorithms have been successfully applied to solve the DSEP problem [14–17]. An algorithm based on the branch exchange technique was developed in [16], while in [17] this technique is improved upon by using a heuristic procedure to obtain better solutions for the network by adding trans-shipment nodes. Several works applied meta-heuristics in the solution of the DSEP problem [18–22]. In [18,19] genetic algorithms are used. A method based on ant colony search was developed in [20], while [21,22] used simulated annealing.

The reliability of the system has been considered using multi-objective approaches for the DSEP problem, as can be seen in [23][26]. In [23,24], evolutionary algorithms were used to solve the DSEP problem, taking into account monetary costs and the system failure index. In [25], multi-objective algorithms, such as NSGA and SPEA, were applied to solve the design of distribution systems. Ref. [26] presented a method to obtain a Pareto front for the multi-objective DSEP problem, employing a dynamic programming approach.

In this paper, a mixed-integer quadratically-constrained programming (MIQCP) model to solve the DSEP problem is proposed. The DSEP model considers the construction/reinforcement of substations, the construction/reconductoring of circuits, the allocation of capacitors banks and the radial topology modification of the system. In contrast with other works on the DSEP problem, the proposed model considers the allocation of fixed capacitors banks in the DSEP problem. As the DSEP problem is a very complex MINLP problem, it is convenient to reformulate it like a MIQCP problem; it is therefore demonstrated that the proposed formulation represents the steady-state operation of a radial distribution system. The MIQCP model is a convex formulation that allows to find the optimal solution of the problem using optimization solvers. In contrast with other MIQCP formulations for the DSEP problem [12], the proposed model uses the original variables of the power flow of the distribution system (voltage magnitudes, current flow magnitudes and power flows). Test systems of 23 and 54 nodes and one real distribution system of 136 nodes were used to show the efficiency of the proposed model in comparison with other DSEP models available in the specialized literature.

The main contributions of this paper are as follows:

1. A novel quadratically-constrained programming model to obtain the steady-state operation of a radial distribution system.
2. A MIQCP formulation for the DSEP problem with the following benefits: (a) a robust mathematical model that finds the same solution of the MINLP model; (b) an efficient computational behavior with conventional MIQCP solvers; (c) convergence to optimality is guaranteed using classical optimization techniques.
3. The DSEP model considers the construction/reinforcement of substations, the construction/reconductoring of circuits, the allocation of capacitors banks and the radial topology modification.

2. The distribution system expansion planning problem

2.1. Steady-state operation of a radial distribution system

The equations that represent the steady-state operation of a radial distribution system are presented in (1)–(4). Note that (1)–(3) are linear, while (4) is non-linear, containing square terms and the product of two variables. These equations are frequently used in the load flow sweep method of radial distribution systems [27,28], and can be used to formulate an MINLP model for the DSEP problem.

$$\sum_{ki \in \Omega_l} \widehat{P}_{ki} - \sum_{ij \in \Omega_l} \left(\widehat{P}_{ij} + R_{ij} \widehat{I}_{ij}^{sqr} \right) + P_i^S = P_i^D \quad \forall i \in \Omega_b \tag{1}$$

$$\sum_{ki \in \Omega_l} \widehat{Q}_{ki} - \sum_{ij \in \Omega_l} \left(\widehat{Q}_{ij} + X_{ij} \widehat{I}_{ij}^{sqr} \right) + Q_i^S = Q_i^D \quad \forall i \in \Omega_b \tag{2}$$

$$V_i^{sqr} - V_j^{sqr} = 2(R_{ij}\widehat{P}_{ij} + X_{ij}\widehat{Q}_{ij}) + Z_{ij}^2 \widehat{I}_{ij}^{sqr} \quad \forall ij \in \Omega_l \tag{3}$$

$$V_j^{sqr} \widehat{I}_{ij}^{sqr} = \widehat{P}_{ij}^2 + \widehat{Q}_{ij}^2 \quad \forall ij \in \Omega_l \tag{4}$$

Considering the following four assumptions, the steady-state operation of a radial distribution system can also be obtained using a quadratically-constrained programming problem, as shown in (5): (a) the real power losses are being minimized in an objective function, which implies that the current flow magnitudes in the branches are also minimized, supposing $R_{ij} > 0$; (b) a radial operation of the distribution system; (c) V_j^{sqr} and $\widehat{I}_{ij}^{sqr}$ are non-negative variables; and (d) the Lagrange multipliers of the second-order cone constraints (5.b) are greater than zero.

$$\left.\begin{array}{ll} \min \sum_{ij \in \Omega_l} R_{ij} \widehat{I}_{ij}^{sqr} & \text{(a)} \\ \text{subject to} & \\ \text{constraints (1)–(3).} & \\ V_j^{sqr} \widehat{I}_{ij}^{sqr} \geqslant \widehat{P}_{ij}^2 + \widehat{Q}_{ij}^2 \quad \forall ij \in \Omega_l & \text{(b)} \end{array}\right\} \tag{5}$$

Note that (4) was re-written like a second-order cone constraint (5.b), that belongs to the set of quadratic constraints [29]. The problem (5) is convex, which makes it possible to find the optimal solution using optimization solvers [30]. In the Appendix section it is demonstrated that, in the solution point of (5), the constraint (5.b) is active and is equivalent to (4). Therefore, solving (5) is equal to solving (1)–(4), thus the quadratically-constrained programming problem (5) effectively represents the steady-state operation of a radial distribution system. In the next subsection this formulation will be used to model the DSEP problem like a MIQCP.

2.2. Mixed-integer quadratically-constrained programming model for the DSEP problem

The DSEP problem is modeled as an MIQCP problem as shown in (6)–(35). Since the proposed model for the DSEP problem considers a static formulation, the solution of the MIQCP model establishes the investments that must to be made in the system, in order to meet future demand and satisfy operational constraints.

$$\begin{aligned} \min \quad & \sum_{i \in \Omega_s} (\kappa_s c_i^s w_i + \alpha f(\tau_s, \lambda) \phi_s c_i^v S g_i^{sqr}) \\ & + \sum_{ij \in \Omega_l} \sum_{a \in \Omega_a} \left(\kappa_l c_{ij,a}^f z_{ij,a} + \alpha f(\tau_l, \lambda) \phi_l c^l R_a I_{ij,a}^{sqr} \right) l_{ij} \\ & + \sum_{i \in \Omega_b} (c^{fx} q_i + c^{un} n_i^{cp}) \end{aligned} \tag{6}$$

subject to

$$\sum_{ki \in \Omega_l} \sum_{a \in \Omega_a} P_{ki,a} - \sum_{ij \in \Omega_l} \sum_{a \in \Omega_a} \left(P_{ij,a} + R_a l_{ij} I_{ij,a}^{sqr} \right) + P_i^S = P_i^D \quad \forall i \in \Omega_b \tag{7}$$

$$\sum_{ki \in \Omega_l} \sum_{a \in \Omega_a} Q_{ki,a} - \sum_{ij \in \Omega_l} \sum_{a \in \Omega_a} \left(Q_{ij,a} + X_a l_{ij} I_{ij,a}^{sqr} \right) + Q_i^S + Q^{cp} n_i^{cp} = Q_i^D \quad \forall i \in \Omega_b \tag{8}$$

$$V_i^{sqr} - V_j^{sqr} = \sum_{a \in \Omega_a} \left[2(R_a P_{ij,a} + X_a Q_{ij,a}) l_{ij} + Z_a^2 l_{ij}^2 I_{ij,a}^{sqr} \right] + b_{ij} \quad \forall ij \in \Omega_l \tag{9}$$