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## An integer programming model for two- and three-stage two-dimensional cutting stock problems

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## ABSTRACT

In this paper, an integer programming model for two-dimensional cutting stock problems is proposed. In the problems addressed, it is intended to cut a set of small rectangular items of given sizes from a set of larger rectangular plates in such a way that the total number of used plates is minimized.

The two-stage and three-stage, exact and non-exact, problems are considered. Other issues are also addressed, as the rotation of items, the length of the cuts and the value of the remaining plates.

The new integer programming model can be seen as an extension of the “one-cut model” proposed by Dyckhoff for the one-dimensional cutting stock problem. In the proposed model, each decision variable is associated with cutting one item from a plate or from a part of a plate resulting from previous cuts (residual plates).

Comparative computational results of the proposed model and of models from the literature are presented and discussed.

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### 1. Introduction

In this paper, an exact approach to two-dimensional cutting stock (2DCS) problems is proposed. The approach is based on the definition of an integer programming model, with a pseudo-polynomial number of variables and constraints, to be optimized directly by a general integer programming solver.

In a 2DCS problem it is intended to cut a set of rectangular items from a set of rectangular plates in such a way that the number of used plates is minimized. The plates are available in (a virtually) infinite number and all have the same dimensions, i.e., the same width and height. A set of items to be cut, grouped by types according to their dimensions (width and height), is given. Each item type is defined by a width, a height and a demand (corresponding to the number of items of the type to be cut).

According to the typology of Wäscher et al. (2007) the problems addressed in this paper are two-dimensional rectangular Single Stock Size Cutting Stock Problems (SSSCSP) with additional constraints related with the types of cuts allowed. In Section 3, the extension to deal with multiple stock sizes (MSSCSP) is also described.

The motivation behind this work lies in the woodcut industry where plates of wood must be cut in pieces (items) to satisfy customer orders. In this industry, due to technological constraints, usually only cuts parallel to the sides of the plate (orthogonal cuts) and from one border to the opposite one (guillotine cuts) are allowed. Furthermore, the number of stages (set of cuts with the same orientation – horizontal or vertical) in which a plate can be cut is also frequently limited to two or three.

In a two-stage problem, the items are obtained by a set of horizontal cuts, dividing the plate in stripes, followed by a set of vertical cuts, separating the different items. If an additional set of horizontal cuts is allowed in order to separate the waste from the items, then the problem is a non-exact two-stage problem (in opposition to the exact case where all the items in a given stripe have the same height). Fig. 1 shows a non-exact two-stage cutting pattern, where the white rectangles correspond to waste and the shaded rectangles to items.

The three-stage problem is also considered in this paper. In this problem, a third set of cuts is allowed: after the plate is cut in stripes (in the first stage), each stripe is then cut in stacks (second stage) and finally the items in each stack are separated by a third set of (horizontal) cuts (third stage). As in the two-stage problem, the exact and non-exact cases are considered. In the latter, an additional set of vertical cuts is allowed for separating the items from the waste. In the former, all the items in a stack must have the same width. In both cases, the restricted version of the three-stage problem is considered, in which, in each stripe, there is always a

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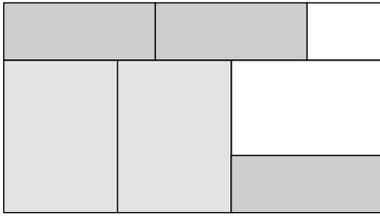


Fig. 1. A non-exact two-stage cutting pattern.

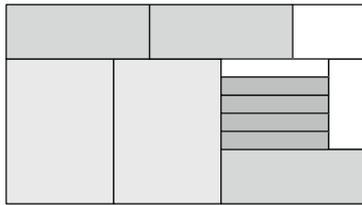


Fig. 2. A non-exact three-stage cutting pattern.

stack with only one item defining its height. See Fig. 2 for an example of a non-exact three-stage cutting pattern.

Previous exact approaches have been proposed for 2DCS problems.

Following their seminal work on the one-dimensional cutting stock problem, Gilmore and Gomory (1965) proposed a column generation algorithm for the two-stage 2DCS problem. As in the one-dimensional case, in the restricted master problem, variables are associated with cutting patterns. The subproblem generates attractive patterns, which, for the two-stage problem, are obtained by solving a sequence of two integer knapsack problems. In Cintra et al. (2008), the subproblem of the column generation algorithm is solved by dynamic programming.

Vanderbeck (2001) also used a column generation based approach for a three-stage 2DCS problem with additional constraints related to the cutting process. The subproblem of generating attractive patterns is also solved by column generation.

Specifically for the two-stage problem, Gilmore and Gomory (1965) also proposed an integer programming model with two sets of decision variables: a set of decision variables related with the definition of the height of the stripes and the other set related with the items included in each stripe. As suggested by Gilmore and Gomory, this model can be solved directly or by applying column generation.

Cutting stock problems are closely related to bin-packing problems. In bin-packing problems there is a strongly heterogeneous assortment of items, i.e., there are many item types with small demands (in the limit, all items are different, being the demands of the item types equal to one). On the other hand, in cutting stock problems, there is a weakly heterogeneous assortment of items, i.e., there is a small number of item types and their demands are large.

Column generation has also been used for bin-packing problems. Two recent approaches were proposed by Pisinger and Sigurd (2007) and Puchinger and Raidl (2007). In the former, the subproblem is solved by constraint programming, allowing to take into account more general patterns, such as non-guillotine cutting. In the latter, the three-stage problem is addressed. The subproblem is solved by a hierarchy of four methods (a greedy heuristic is first applied followed by a genetic algorithm, which is followed by an exact method for a simplified version of the (sub)problem, and, only if no attractive pattern is obtained by the previous methods, is the (sub)problem solved exactly).

Compact integer programming models have also been proposed for bin-packing problems. Since the number of variables and constraints is polynomial with respect to the size of the instance, the implementation step is reduced to inputting the model in an integer programming solver. That is the case of the models proposed in Lodi et al. (2004) for the two-stage problem and extended by Puchinger and Raidl (2007) to the three-stage problem. In both cases, decision variables are related to the assignment of the items to bins, stripes or stacks. Typically the linear relaxation of these “assignment” models provide lower bounds that, in general, are worse than the ones given by the linear relaxation of the models mentioned so far (Gilmore and Gomory, 1965; Cintra et al., 2008; Vanderbeck, 2001; Pisinger and Sigurd, 2007).

A cutting problem related to the one addressed in this paper is the constrained two-dimensional cutting stock problem. In this problem, it is intended to maximize the profit of the items (available in finite number) cut from a single plate. In Belov and Scheithauer (2006), a branch-and-cut-and-price algorithm is proposed for the two-stage non-exact constrained two-dimensional cutting stock problem. In each branch-and-price node, Chvátal-Gomory cuts and Gomory mixed-integer cuts are used to improve the quality of the linear programming relaxation solved. For the same problem, Hifi and Roucairol (2001) proposed exact and heuristic algorithms and Lodi and Monaci (2003) proposed two integer linear programming models. More recently, Cui (2008) presented a branch-and-bound procedure combined with dynamic programming for the homogenous three-stage constrained two-dimensional cutting stock problem.

For a survey on two-dimensional packing problems, the interested reader is referred to Lodi et al. (2002). For more general surveys on cutting and packing problems see Wäscher et al. (2007) and Dyckhoff et al. (1997). There is also a categorized database of publications on cutting and packing available at the “EURO Special Interest Group on Cutting and Packing” site (<http://paginas.fe.up.pt/~esicup/>).

In this paper, an approach to obtain optimal solutions to two-stage and three-stage 2DCS problems, which is not based on column generation, is proposed. The model is an extension of the one-cut model for the one-dimensional cutting stock problem proposed by Dyckhoff (1981). The approach is based on an integer programming model that can be solved by a general integer programming solver, taking advantage of the efficiency and robustness that (mixed-) integer programming solvers have achieved in recent years. For example, in the software used in this paper for the computational tests – CPLEX 11.0 (see Ilog, 2007) – several classes of valid inequalities and heuristics are implemented to improve the (lower and upper) bounds during the search of the branch-and-bound tree.

The model inherits the possibility of modeling the features already pointed out by Dyckhoff, as multiple types of pieces (plates) and the value of the pieces (plates) for future use. Furthermore, there is a feature of the proposed model, which is very relevant in practice but is not usually taken into account, which is modeling the length (or time) of the cuts needed to obtain all the final items. This issue as well as an extension for the rotation of pieces will be addressed after the presentation of the model.

This paper is organized as follows. In the next section, the integer programming model for the two-stage and three-stage, exact and non-exact, 2DCS problems is introduced. Upper bounds on the number of constraints and on the number of variables of the model are given. The model is based on the enumeration of all the different cuts and residual plates, for which an algorithm is described. Section 2 ends with a small example. In Section 3, extensions of the developed model are discussed and proposed. Computational results are reported and analyzed in Section 4. In Section 5, the main conclusions of this work are drawn.

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