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Discrete Optimization

New mixed integer-programming model for the pickup-and-delivery problem with transshipment

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ABSTRACT

In recent years, many important real-world applications are studied as "rich" vehicle routing problems that are variants and generalizations of the well-known vehicle routing problem. In this paper we address the pickup-and-delivery version of this problem and consider further generalization by allowing transshipment in the network. Moreover, we allow heterogenous vehicles and flexible fleet size. We describe mixed integer-programming formulations for the problem with and without time windows for services. The number of constraints and variables in the models are bounded by polynomial size of the problem. We discuss several problem variants that are either captured by our models or can be easily captured through simple modifications. Computational work gave promising results and confirms that transshipment in network can indeed enhance optimization.

1. Introduction

For a given set of vehicles and customers, the classical *Vehicle Routing Problem* (*VRP*) seeks to find a set of minimum-cost routes of the vehicles for visiting all customers from a central vehicle depot. The *Pickup-and-Delivery Problem* (*PDP*) is a generalization of the VRP in which the vehicles are required to service customer requests for picking up and delivering transport loads; the pickup and delivery customers are usually paired. An optimal solution is a set of minimum-cost vehicle routes for servicing all customer requests where the variable costs are associated typically with the distances between the customer locations and the fixed costs are incurred for use of the vehicles.

Many real-world logistics and transportation issues entail time windows for services; therefore, addressing also the time windows is a natural extension of the VRP. Inclusion of time windows thus defines an important class of VRP problems commonly known as the *Vehicle Routing Problem with Time Windows* (*VRPTW*); the *Pickup-and-Delivery Problem with Time Windows* (*PDPTW*) is its analogous counterpart in which both pickup and delivery services have time windows imposed.

The PDP and the PDPTW together address an enormous suite of real-world applications (Toth & Vigo, 2002). Common examples abound in transportation and logistics management such as routing and scheduling of public transports, transportation-on-demand, emergency transportation of goods and personnels, as well as logistics in humanitarian and disaster management (Christiansen, Fagerholt, Nygreen, & Ronen, 2007; Golden, Assad, & Wasil, 2002; Macharis & Bontekoning, 2004; Powell, Bouzaiene-Ayari, & Simao, 2007).

In this paper we consider a generalization of the PDP by focusing closely on the real-world needs and practices for efficient transport systems. As transportation of people and goods is vital for the economy and the quality of life, efficient transport systems are essential for promoting sustainable growth of economic activities and global competitiveness. Indeed, a recent white paper on transport by the European Commission emphasized urgent needs to adopt innovative transport solutions and optimize performance of transport systems through efficient management of multimodal logistic chains and linking long-haul with last-mile transportation coverage (Roadmap to a Single European Transport Area, 2011).

In order to address these issues and facilitate transfers as well as modifications of the transport loads carried by the vehicles, we consider the notion of transshipment in PDP to get the *Pickup-and-Delivery Problem with Transshipment* (*PDPT*) that has designated transshipment locations in the underlying network. The PDPT is thus a generalization of the PDP for allowing potentially more routing options as well as improving capacity utilization of the vehicles; therefore, it is also a *NP*-hard combinatorial optimization problem (Savelsbergh & Sol, 1995).

For a general description of the PDPT, we are given a set of vehicles, locations of the vehicle origin and final depots, a set of locations of customer requests for transporting loads from the pickup to the delivery locations, and the network connecting the locations.

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Each vehicle is initially located at an origin vehicle depot and its route through the network must end at some final vehicle depot. Customers along with the associated requests for either pickup or delivery of the transport loads are represented by their locations in the network. Unlike in the PDP, the network in the PDPT can have designated transshipment locations where vehicles may stop for transfers and adjustments of their transport loads, the drivers may switch their vehicles or get release times for conforming to policy-related matters, fresh or rested drivers may replace the tired drivers, etc. Typically, similar to the PDP, the optimization aspect of the PDPT also seeks to service all customer pickup-and-delivery requests at minimum total cost where the variable costs are associated typically with the distances between the customer locations and the fixed costs are incurred for use of the vehicles.

This general definition of the PDPT allows multiple origin and final depots for the vehicles, a fleet of heterogenous vehicles having different load-carrying capacities, and unlimited transshipments or transfers of the requested transport loads at the designated transshipment locations. Moreover, distinct customer requests may have the same pickup or delivery locations – i.e., a pickup or delivery location can be involved with more than one customer request. Furthermore, customer requests may or may not be split for carrying by different vehicles from the pickup to the delivery locations. As such, many interesting problem variants can arise and the objective function for optimization can be defined in many different ways – we discuss them in Section 5.

Allowing flexibility of transshipment in the network poses additional challenges for modeling the generalized VRP as spatial and time operations involving the vehicles must be properly matched and synchronized. The model must ensure proper representation of the network for facilitating assignments of the customer requests to potentially various modes of transportation. The model must also incorporate many inherent restrictions typical in multimodal transportation, such as: certain regions may support only rail transportation while other regions may allow land as well as sea options; certain transport loads may only be carried by specific transporters (e.g., crude petroleum in large ocean-going tankers); transportation of hazardous materials through any urban areas may be required to use specialized vehicles as well as specific roads; perishable and pharmaceutical products may only be transported in time-critical vehicles having refrigeration capabilities; or, certain transport loads may be "split" and shared by more than one vehicle to complete the task of transportation.

For addressing such requirements in the real-world applications, much closer attention to detail is necessary for making use of the entire transport system consisting of the land, sea and air modes along with all provisions for any differentiated short-haul and long-haul transport options. For short-haul transportation, transshipment typically allows more flexibility without incurring much additional costs while potentially decreasing the quality of service for inconvenience to passengers or risks of deterioration of goods. However, this may also entail synchronization aspects which are often difficult to satisfy in practice. For long-haul transportation, transshipment typically relates to multimodal transportation which generally involves additional cost, time and storage.

Having flexibility in such systems has become increasingly important in the current globalized as well as competitive economy and this essentially provides impetus for multimodal planning and exploiting all potential options for optimization. However, fruitful research in this domain has been deemed to be in its early stages (Macharis & Bontekoning, 2004) and, to the best of our knowledge, comprehensive work addressing the relevant needs and issues within the framework of optimization is quite recent (Drexl, 2012a, 2012b, 2013). The author has focused on the PDPT and other generalizations of the PDP for numerous real-world scenarios – we will discuss them with literature review in Section 2.

In this paper, we address the PDPT with and without the time windows for services and present *Mixed Integer Programming* (*MIP*) models for solving the problems. We allow vehicle fleet to be heterogenous and its size to be flexible. The number of constraints and variables in the models are shown to be bounded by polynomial size of the problem. We discuss several problem variants that are either captured by our models or can be easily captured through simple modifications. Computational work gave promising results and confirms that transshipment in network can indeed enhance optimization.

The rest of the paper is organized as follows. We discuss literature relevant to the PDPT in Section 2. In Section 3, we provide a simple example to illustrate the impact of transshipment. We present MIP models for the PDPT in Section 4, followed by a discussion on related problem variants in Section 5. We provide computational analysis in Section 6 and concluding remarks in Section 7.

2. Literature review

Survey of the VRP that includes the PDP can be found in Assad (1988) and Desrosiers and Dumas (1988) and a general overview of the PDP along with the solution methods then available is given in Savelsbergh and Sol (1995). Specific surveys addressing these two broad problem classes are found in Desaulniers, Desrosiers, Erdmann, Solomon, and Soumis (2002) and Mitrović-Minić (1998) while a fairly comprehensive and recent survey of many variants of the VRP including the PDPT is found in Parragh, Doerner, and Hartl (2008). Classification of related problems including the PDPT is given in Berbeglia, Cordeau, Gribkovskaia, and Laporte (2007). Recent advances in the exact solution methods for solving the problems are reviewed in Cordeau, Desaulniers, Desrosiers, Solomon, and Soumis (2002) and Cordeau and Laporte (2007) and dynamic PDPT is addressed in Bouros, Sacharidis, Dalamagas, and Sellis (2011).

To the best of our knowledge, only a handful of papers have so far addressed the PDPT. In three seminal papers (Drexl, 2012a, 2012b, 2013), the author underlined key hurdles for tackling various synchronization aspects including the PDPT and several related problem variants. He introduced different classes of the problems for various vehicle types such as lorries, tractors, trailers and semi-trailers. He has highlighted practical issues arising from a real-world application as a backdrop: Milk collection business in rural Germany. He also highlighted the key hurdles confronting the efforts for developing proper solution methodologies and stressed the needs for focusing on synchronization of the vehicles regarding spatial, temporal, and load-carrying aspects.

A complete MIP model for the PDPT is given in Cortés, Matamala, and Contardo (2010). The authors considered passenger transportation and used decision variables for the arcs as well as the nodes in the underlying network. Their model provides options to passengers for transfer from one vehicle to another at the designated transfer nodes. For each transfer node, they split the node into two and connect the two parts by an arc to explicitly capture the arrival and the departure of a vehicle – thus handling precedence relationships for the transfers of the passengers. Each vehicle is allowed to use a transfer node at most once but via duplication of the transfer nodes in the network, a vehicle may use a transfer node more than once. In total, their model has 23 different sets of constraints including standard arc variables for the vehicle flows and binary node variables to explicitly track the passengers along their entire routes through the network – they do not use flow variables for the flow of the passengers. They presented a branch-and-cut algorithm based on Benders decomposition to solve the model and gave computational results. However, the size of the problem instances addressed is small: