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# A bi-objective programming model for designing compact and balanced territories in commercial districting

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ABSTRACT

In this paper, we address a territory design problem arising from a bottled beverage distribution company. We propose a bi-objective programming model where dispersion and balancing with respect to the number of customers are used as performance criteria. Constraints such as connectivity and balancing with respect to sales volume are considered in the model. Most of the work in territory design has been developed for single-objective models. To the best of our knowledge, this is the first multi-objective approach for this commercial territory design problem, and in particular, for territory design with connectivity constraints. We propose an improved $\varepsilon$-constraint method for generating the optimal Pareto front. Empirical evidence over a variety of instances shows that the improved method is well suited for finding optimal Pareto fronts with no more computational effort than the traditional method. Instances of up to 150 units and 6 territories are solved in relatively short amount of time. For this problem, the improved method finds practically the same fronts than those found by the traditional $\varepsilon$-constraint method. In addition, we observe that when the firm reduces the tolerance in the imbalance of sales volume the efficient fronts change and when the number of territories increases, the balance with respect to the number of customers becomes harder to achieve.



## 1. Introduction

In general, distribution firms have complex product distribution networks which are formed by thousands of sales points. In this industry there are many interesting problems from the logistic point of view that may appear in different stages of the decision process. For instance, when a firm is starting, a first problem could be where to locate the warehouses and/or distribution centers. After that, in order to provide efficient service and to reduce the total costs (i.e., production, stock, and distribution costs) some questions such as how many products need to be produced, and how to deliver the products to the final customer, need to be answered. This work is focused on the study of a problem that arises in a stage previous to the product routing and is motivated by a real-world application from a beverage distribution firm in the city of Monterrey, Mexico. The firm wants to divide the total number of city blocks into a specific number of groups according to some planning criteria. This partition has the objective of giving support to the decision maker when she or he designs the distribution routes and when she or he makes the workload distribution. In addition, the partition permits a more efficient management of marketing offers as it reduces the number of unsatisfied customers by applying special offers in each territory. This means that we are contributing to better route design during the routing process due to the compactness (minimum dispersion) of the territories. In addition, we provide support to the decision maker for elaborating the marketing

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plan and for making the best workload and resource distribution. The latter is possible because the territories are balanced with respect to both number of customers and sales volume.

This problem belongs to the family of districting problems. There has been a significant amount of work in the territory design literature addressing many different kinds of applications such as political, sales, school, services, and commercial districting, to name the most significant. Among the most relevant works one can find Hess et al. (1965), Fleischmann and Paraschis (1988), Hojati (1996), Garfinkel and Nemhauser (1970), Mehrotra et al. (1998), Bozkaya et al. (2003), and Kalcsics et al. (2005). In practically all of these works, the authors consider single-objective models. Among the very few works dealing with multi-objective districting problems we find Bowerman et al. (1995), Scott et al. (1996), Guo et al. (2000), Wei and Chai (2004), Tavares-Pereira et al. (2007), and Ricca and Simeone (2008).

Bowerman et al. (1995) present a multi-objective approach for solving a school bus routing problem. They proposed a heuristic technique that first groups students into clustering using a multi-objective districting algorithm. After that, a school bus route and the bus stops for each cluster are generated by using a combination of a set covering procedure and a traveling salesman problem procedure. They report experimental results for a real-world instance in Wellington County, Ontario. The districting algorithm considers four objectives: minimizing the number of routes, minimizing the length of the routes, load balancing, and compactness of the routes. The three last criteria are placed in a weighted objective function where the number of routes is the dominant objective, i.e., a solution with fewer routes is always favored over a solution with more. Different plans were designed using different sets of weights over the optimization criteria.

Scott et al. (1996) make a multi-objective analysis of school districting in a case study from Connecticut, USA. They propose a mixed-integer goal programming model where the goal constraints are to minimize disparities in: minority enrollments, grand-list/student ratios, student–teacher ratios, and overall enrollment. The number of districts is not fixed and the contiguity criterion is not formulated in an explicit way. Experimental work using different weighting scenarios reveals that the traditional distance-minimizing or transportation-minimizing objectives are in conflict with all other aims of equity and quality of educational opportunities.

Guo et al. (2000) propose a multi-objective zoning and aggregation tool (MOZART). MOZART is an integration of a graph partitioning engine with a Geographic Information System (GIS) through a graphical user interface. They illustrate the performance of MOZART by solving two zoning problems from three government local areas in Victoria: Kingston, Bayside, and Glen Eira. The first part of their experimental work is done by taking into account a single objective of equality in population size. In contrast, in the second part of their experimental work, both equity in population and compactness are treated as objective functions. They report a case with 577 census collection districts and 20 zones. The inclusion of compactness as the second zoning objective yields zones with better shapes.

Wei and Chai (2004) present a multi-objective hybrid metaheuristic approach for a GIS-based spatial zoning model. Their heuristic procedure is a combination of tabu search and scatter search. They show the procedure performance by solving a political districting problem with 55 basic units and 3 districts. Equity in population, compactness, and socio-economic homogeneity are treated as objectives.

Tavares-Pereira et al. (2007) study a multi-objective public service districting problem. They consider multiple criteria such as location of the zone with respect to the network, mobility structure within a zone, zone corresponding to administrative structures, centers of attraction in the zone, social nature and geographical nature. They propose an evolutionary algorithm with local search and apply it to a real-world case of the Paris region public transportation. They discuss results for bi-objective cases considering different criteria combinations.

Ricca and Simeone (2008) address a multiple criteria political districting problem. Such criteria are connectivity, population equality, compactness and conformity to administrative boundaries. They transform the multi-objective model into a single-objective model using a convex combination of three objective functions (inequality, noncompactness, and nonconformity to administrative boundaries); connectivity is considered as a constraint. They compare the behavior of four local search metaheuristics: descent, tabu search, simulated annealing, and old bachelor acceptance. The application is performed over a sample of five Italian regions where old bachelor acceptance produces the best results in most of the cases.

The state of the art on territory design reveals the following facts. Very few works address multi-objective models and all of these are basically heuristic techniques for obtaining approximate Pareto fronts. To the best of our knowledge our work is the first to provide a method for obtaining efficient fronts to bi-objective territory design problems. Single-objective versions of the commercial territory design problem addressed in this work are due to Ríos-Mercado and Fernández (2009), Caballero-Hernández et al. (2007), and Segura-Ramiro et al. (2007). In particular, our work can be seen as the bi-objective extension to the model developed in Segura-Ramiro et al. (2007).

Our work comprises both the development of a bi-objective optimization model and an exact optimization procedure for finding efficient solutions in the sense of Pareto. The solution procedure is based on one of the most important scalarization techniques in multi-objective programming, the $\varepsilon$-constraint method. We implement two alternatives of this method: the traditional $\varepsilon$-constraint method ($\varepsilon$CM) which guarantees obtaining weakly efficient solutions and a modified version of the $\varepsilon$-constraint method (I$\varepsilon$CM) in which we include slack variables to guarantee efficient solutions. The last technique was recently proposed by Ehrgott and Ruzika (2008) in the improved $\varepsilon$-constraint method. Our computational work reveals the effectiveness of the proposed approach as it was able to obtain Pareto fronts to a large class of problem instances in a relatively small computational effort. This is, to the best of our knowledge, the first exact method for a multi-objective territory design problem with connectivity constraints.