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# A two-stage stochastic programming model for the parallel machine scheduling problem with machine capacity

Talal Al-Khamis *, Rym M'Hallah

*Department of Statistics and Operations Research, Kuwait University, P.O. Box 5969, Safat 13060, Kuwait*

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ABSTRACT

This paper proposes a two-stage stochastic programming model for the parallel machine scheduling problem where the objective is to determine the machines' capacities that maximize the expected net profit of on-time jobs when the due dates are uncertain. The stochastic model decomposes the problem into two stages: The first (FS) determines the optimal capacities of the machines whereas the second (SS) computes an estimate of the expected profit of the on-time jobs for given machines' capacities. For a given sample of due dates, SS reduces to the deterministic parallel weighted number of on-time jobs problem which can be solved using the efficient branch and bound of M'Hallah and Bulfin [16]. FS is tackled using a sample average approximation (SAA) sampling approach which iteratively solves the problem for a number of random samples of due dates. SAA converges to the optimum in the expected sense as the sample size increases. In this implementation, SAA applies a ranking and selection procedure to obtain a good estimate of the expected profit with a reduced number of random samples. Extensive computational experiments show the efficacy of the stochastic model.

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## 1. Introduction

Parallel-machine scheduling problems are important from the theoretical and practical viewpoints. They are the corner stone of many elaborate theoretical models and play a crucial role in various applications such as manufacturing, communication, and computer science. Since the pioneering work of Mc Naughton [19], many optimization based methods have been suggested for this scheduling problem. As can be inferred from the comprehensive reviews of Mokotoff [18] and Baker and Scudder [2] for the identical machine case and Li and Yang [14] for the non-identical machine case, most of these methods (e.g. [20,25,27]) assume deterministic operational parameters for the problem (such as processing times, machines' capacities, and jobs' due dates).

Since most parallel-machine parameters are random in nature, researchers are focusing on the stochastic version of this problem. For instance, Cai et al. [3] study the identical parallel-machine problem where the processing times are random and the objective is to minimize the expected discounted holding cost. Arnaout et al. [1] address the unrelated parallel machine batch scheduling problem with sequence dependent setup times and stochastic processing times and setup times. They consider minimizing the total weighted mean completion time. They present new heuristics for the problem and compare their performance to existing ones using simulation. Gu et al. [9] present a quantum genetic algorithm to solve the stochastic earliness and tardiness parallel-machine scheduling problem. Skutella and Uetz [23] derive the first constant-factor approximation algorithms for the parallel identical machine scheduling problems where jobs are subject to precedence constraints and release dates, the processing times are governed by independent probability distributions, and the objective is to minimize the expected value of the total weighted completion time. Cai and Zhou [4] address the stochastic identical parallel machines scheduling problem in which a set of independent jobs are to be processed under a common deadline. Each job has a random processing time, drawn from an arbitrary distribution, and an exponentially distributed deadline. Each machine is subject to stochastic breakdowns, occurring according to a Poisson process. The objective is to minimize the expected total cost of earliness and tardiness, where the cost of an early job is a general function of its earliness whereas the cost of a tardy job is a fixed charge.

Most of the above suggested models are suitable when the number of values (herein referred to as scenarios) the uncertain parameters can assume is limited. When there are many uncertain parameters, the number of scenarios becomes unmanageably large, and determining the optimal solution of the stochastic problem becomes difficult and very time consuming. This paper deals with a stochastic parallel machine problem whose number of scenarios tends to infinity. Specifically, it addresses the case

* Corresponding author. Tel.: +965 997 70875.
*E-mail address:* alkhamis@kuc01.kuniv.edu.kw (T. Al-Khamis).

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where the due dates are uncertain, and the objective is to determine the machines' capacities that maximize the expected net profit; i.e., the sum of the machines' costs and the expected profit resulting from on-time jobs.

This problem occurs frequently in many real world manufacturing industries. The planning engineer has to lease resources prior to knowing the due dates of the jobs the plant has to process. Then, given a set of resources and the due dates of the jobs, the planning engineer chooses the jobs to be processed. The problem is more challenging as the due dates are not known with a high degree of precision, and cannot really be assumed deterministic for all practical purposes. For instance, in make to order apparel, food, furniture and petro-chemical industries, clients specify a due date; however, they do not necessarily abide to it: they may end up requesting the item either earlier or later but seldom on the specified due date. Even though they are the source of uncertainty, clients do not react well in either situation. Evidently, this problem occurs in other settings such us truck pick up and delivery, just in time environments, health care, etc.

This problem was inspired from a real world application. This latter was brought to our attention by a senior manager of a petroleum service company which was conducting many electrical engineering construction projects concurrently on an oil production site. The company receives requests for preliminary work from its projects' managers. The work is to be undertaken by its civil engineering division before the electrical engineering tasks are initiated. It involves the use of heavy duty equipments including cranes, drills, earth moving trucks, excavators, reamers, rigs, etc. Each request specifies the task to be undertaken, the equipments it requires, the duration of the task, and when the task should be completed so that the electrical engineering components of the projects can be started on time. The company knows that the specified due dates are random variables; each following a known uniform distribution. It has to decide (i) whether to lease/buy additional units for each type of equipment or subcontract parts of the tasks and (ii) the utilization level of each unit of equipment. A further complicating aspect is that the company has to obtain a work permit for each unit of equipment prior to the beginning of the projects. The work permit explicitly states the number of hours the unit will be available, where availability is governed by local environmental, labor, safety and security laws. A straightforward solution could be to have an excessively large number of units per equipment type. However, this suggestion is infeasible since these machines are in high demand not only locally but also worldwide. In fact, their sales are at all time high with almost all manufacturers struggling to meet demand [29]. In addition, their unnecessary presence on site could lead to huge fines. Therefore, their use (in particular, their optimal operational capacity) has to be well defined. The senior manager did not oppose the idea of optimizing the use of each type of equipment individually; in particular the use of rigs for preparing the infrastructure for cathodic protection installations.

This scheduling problem is modeled as a two-stage stochastic program under uncertainty, where the decision variables are partitioned into two sets. The first stage variables correspond to the machines' capacities which are determined by the planning engineer before the uncertain parameters, which are the due dates of the jobs, become known. Once the random events (the due dates) present themselves, the planning engineer can select the value of the second stage variables which correspond to the on-time jobs. The objective is to choose the first stage variables such that the expected net profit is maximized. The expected net profit is the sum of the cost of operating the machines at their optimal capacities given by the first stage variables and the resulting expected profit brought about by the on-time jobs (which are the second stage variables).

The two-stage stochastic model is herein solved via a sample average approximation (SAA) scheme which uses only a subset of randomly sampled scenarios to represent the full scenario space. It chooses, for each sampled scenario, the best machine capacities. Then, it aggregates this information and obtains an estimate of the optimum. An important theoretical justification for SAA is that its solution converges to a solution of the true problem in the expected sense as the number of sampled scenarios increases [17,21]. Herein, SAA incorporates a ranking and selection (RS) procedure that reduces the number of samples to be considered.

The proposed solution approach is innovative in many aspects. To our knowledge, SAA was never applied to the parallel-machine stochastic scheduling problem at hand. In addition, embedding RS into SAA makes the approach better adapted to parameters of the problem (i.e., the due dates' distributions and the machines' capacities); thus, helps the approach avoid issues resulting from both over and under sampling. Finally, our approach uses an exact method to solve the second stage problems to the opposite of many applications which resort to approximately solving them.

Section 2 formulates the problem as a two-stage stochastic model, pinpoints its difficulties, and sketches a solution strategy. Section 3 details the proposed approach. Section 4 illustrates the complexity of solving the stochastic problem using other methods and investigates the performance of the proposed one. Finally, Section 5 summarizes the paper.

## 2. Model

Specifically, consider a stochastic scheduling problem with $m$ identical parallel machines and $n$ jobs. Each machine $i$, $i=1, \ldots, m$, is characterized by its capacity $x_i$, expressed in terms of time units machine $i$ is available, and its unit cost $-c_i$, where $c_i>0$, and the negative sign reflects a cost. There is a finite set $\chi$ having $|\chi|$ machines' capacities $\mathbf{x}=(x_1, \ldots, x_m)$, where $|\chi|$ is small enough to allow complete enumeration of all combinations of machines' capacities. The plant receives an order of $n$ jobs where job $j, j=1, \ldots, n$, is characterized by its processing time $p_j$, its weight $w_j$, and its random due date $D_j$. The due date $D_j$ of job $j$ follows an arbitrary discrete distribution with mean $\mu_{D_j}$. The due dates are independent. It is assumed hereafter that all problem parameters are integer.

The two-stage stochastic model of this scheduling problem proceeds as follows. The first stage (FS) determines the optimal machine capacities $\mathbf{x}^*=(x_1^*,\ldots,x_m^*)\in\chi$, prior to knowing the true values of the uncertain due dates of the $n$ jobs. The optimal machine capacities are those that yield the maximum expected net profit $z^*=z(\mathbf{x}^*)$. This latter sums the cost of operating the machines at their respective capacities $x_i^*$, $i=1, \ldots, m$, and the expected profit induced by on-time jobs when the machines' capacities are $\mathbf{x}^*$. That is,

$$(FS)\quad \begin{cases} z^*=z(\mathbf{x}^*):=\max \sum_{i=1}^{m} -c_i x_i + E[Q(x_1,\ldots,x_m,D_1,\ldots,D_n)] & \text{(a)} \\ \text{s.t. } (x_1,\ldots,x_m)\in\chi, & \text{(b)} \end{cases} \tag{1}$$

where $E[Q(x_1,\ldots,x_m,D_1,\ldots,D_n)]$ is taken with respect to the distributions of the random due dates $D_1, \ldots, D_n$.

The second stage (SS), or recourse problem, determines $q(x_1,\ldots,x_m, d_1,\ldots,d_n)$, the optimal weight of on-time jobs for given first stage decision variables $(x_1,\ldots, x_m)$ and a scenario $(d_1,\ldots,d_n)$ of

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