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Discrete Optimization

Mathematical programming modeling of the Response Time Variability Problem

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ABSTRACT

The Response Time Variability Problem (RTVP) is a scheduling problem that has recently been defined in the literature. The RTVP has a broad range of real-life applications from manufacturing to services and information technology. A previous study developed a position exchange heuristic to apply to initial sequences for the RTVP, and a MILP (Mixed Integer Linear Programming) to obtain optimal solutions with a practical limit of 25 units to be scheduled. This paper aims to improve the best mathematical programming model developed thus far in order to solve larger instances up to 40 units to optimality. The contribution of this paper is 4-fold: (i) larger instances can be solved to optimality by the off the shelf standard software; (ii) the new optimal solutions of the RTVP can be used to compare the results of heuristic procedures; (iii) the importance of modeling is demonstrated, as well as the huge impact that reformulation, redundant constraints and the elimination of symmetries have on the efficiency of MILPs is clearly established; finally (iv) a challenge to develop a customized optimization algorithm to rival the MILP solution efficiency for the RTVP is put forward.

1. Introduction

Corominas et al. (2007) have recently introduced the Response Time Variability Problem (RTVP) to model a broad range of real-life situations that occur whenever events, jobs, clients or products need to be sequenced so as to minimize the variability of the time they wait for their next turn in obtaining the resources they need to advance. For example, it can be used in the automobile industry to sequence the models to be produced on a mixed-model assembly line (Monden, 1983).

This problem does not only occur in the manufacturing industry, but also in computer multi-threaded systems and network servers (e.g., Waldspurger and Weihl, 1995). For example, the data to be sent by an asynchronous transfer mode network is divided into cells of a fixed size. There are voice and/or video applications whose data cells must be regularly sequenced in the stream.

Other contexts in which the RTVP appears are the periodic machine maintenance problem (Wei and Liu, 1983; Anily et al., 1998; Bar-Noy et al., 2002), and the scheduling of waste collection (Herrmann, 2007). Hermann came up with the RTVP while working with a healthcare facility that needed to schedule the collection of waste from waste collection rooms throughout the building. Based on data about how often a waste collector had to visit each room and in view of the fact that different rooms require a different number of visits per shift, the facility manager wanted these visits to occur as regularly as possible so that excessive waste would not collect in any room. For instance, if a room needed four visits per eight-hour shift, it would ideally be visited every two hour. The scheduling of advertising slots for television (Bollapragada et al., 2004; Brusco, 2008) leads to a problem closely related to RTVP and also does the design of sales catalogs (Bollapragada et al., 2004).

These real-life problems are usually considered as distance-constrained scheduling problems (Han et al., 1996; Dong et al., 1998). Although the main objective of the distance-constrained problem and the RTVP is to find as regular a sequence as possible, the advantage of the RTVP is that it will always come up with a feasible solution, contrary to the distance-constrained problem.

The abovementioned applications are examples of a very common situation, in manufacturing and in services, in which a resource must be used successively by different units and it is important to schedule them in such a way that the different types of units share the resource in some fair manner (see Kubiak (2004) who provides an extensive overview on fair sequences). The RTVP proposes a new universal measure of fairness: to minimize the variability of the distance (measured, for example, in number of slot times) between any two consecutive units of the same product (event, job or client); i.e., to have the distances between any two given consecutive units of the same product as constant as possible. Several other measures have been proposed for the fairness of the sequence of models on assembly lines,

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either based on the difference between ideal and actual productions (Miltenburg, 1989; Kubiak, 1993; Steiner and Yeomans, 1993) or on the difference between ideal and actual production dates (Inman and Bulfin, 1991; Bautista et al., 1997). The new measure of fairness is easier to understand by practitioners, since it only uses a simple concept: the distance. Moreover it has the characteristic that the value of the measure does not depend on the position of those products with only one unit to be sequenced.

The RTVP is quite simple to formulate but rather difficult to solve to optimality. Let n be the number of products/jobs/messages (in this paper we will only use the term "product"). Let d_i be the number of units of product i ($i = 1,\dots,n$) to be scheduled in a sequence $s = s_1 s_2 \cdots s_D$ of length $D (D = \sum_{i=1}^{n} d_i)$, i.e., with D positions. For all types of product with $d_i \geqslant 2$, let t_k^i be the distance between the position of units $k + 1$ and k of product i (i.e., the number of positions between them, where the distance between two consecutive positions is considered equal to 1). Let us assume that s_1 immediately follows s_D (i.e., it is a circular sequence). Therefore, $t_{d_i}^i$ is the distance between the first unit of product i in a cycle and the last unit of the same product in the preceding cycle. Let $\bar{t}_i = D/d_i$ be the average distance between two consecutive units of product i. For all models i in which $d_i = 1$, t_1^i is equal to $\bar{t}_i$. The objective is to minimize $RTV = \sum_{i=1}^{n} \sum_{k=1}^{d_i} (t_k^i - \bar{t}_i)^2$, which is equal to minimize $RTV = \sum_{i=1}^{n} d_i \cdot Var_i$, where $Var_i = \frac{1}{d_i} \cdot \sum_{k=1}^{d_i} (t_k^i - \bar{t}_i)^2$ is the variance of the distances between consecutive units of product i.

Corominas et al. (2007) present the Response Time Variability Problem, study its computational complexity and prove that the RTVP is NP-complete. They propose an optimization algorithm for a two-product case. In order to solve the RTVP to optimality they consider a special case of the quadratic assignment problem recast as a quadratic integer programming (QIP) problem. The QIP is linearized using three different techniques, but the practical limit for getting optimal solutions with the best MILP obtained, is 25 units to be scheduled (i.e., $D = 25$). Finally, a simple position exchange heuristic is presented for application to some greedy initial sequences and a computational experiment using the heuristic is reported.

This paper analyzes some special features of the RTVP and presents some new ideas for improving the MILP model used in Corominas et al. (2007) to solve the RTVP optimally. The objective of this paper, therefore, is to improve the best mathematical programming model presented in the literature thus far in order to solve larger instances to optimality. The contribution of this paper is 4-fold: (i) larger instances can be solved to optimality for a problem that has numerous real-life applications; (ii) the new optimal solutions of the RTVP can be used to compare the results of heuristic procedures, which are needed to solve medium-large instances; (iii) the importance of modeling is demonstrated, as well as the huge impact that reformulation, redundant constraints and the elimination of symmetries have on the effectiveness of MILPs; finally (iv) a challenge to develop a customized optimization algorithm (e.g. branch and bound) to rival the MILP solution efficiency for the RTVP is put forward.

The remainder of the paper is set out as follows: first, Section 2 presents the terminology, the best MILP model presented in the literature and a lower bound on the value of the objective function. Section 3 introduces the ideas for improving the MILP. Section 4 presents the results of the subsequent computational experiment. Finally, Section 5 is devoted to conclusions and possible venues for future research.

2. Terminology, initial MILP and a lower bound

This section presents the main terminology that will be used in this paper. It explains the MILP for the RTVP presented in Corominas et al. (2007), referred to as MILP-0 in this paper, and describes a lower bound on the value of the objective function *RTV* given by Corominas et al. (2007).

Data:

n	number of products ($i = 1,\dots,n$)
D	number of positions in the sequence
d_i	number of units of product i ($i = 1,\dots,n$) to be scheduled; it is assumed that $\sum_{i=1}^{n} d_i = D$
$\bar{t}_i$	average distance between two consecutive units of product i: $\bar{t}_i = D/d_i$ $(i = 1, \dots, n)$
$G1$	set of products with multiple copies, that is $d_i \geqslant 2$: $G1 = \{i—d_i \geqslant 2\}$
UB_i	upper bound on the distance between two consecutive units of product i: $UB_i = D - d_i + 1$ $(\forall i \in G1)$
E_{ik},L_{ik}	the earliest and the latest position that can be occupied by unit k of product i: $E_{ik} = k$ and $L_{ik} = D - d_i + k$ ($i = 1,\dots,n$; $k = 1,\dots,d_i$)
H_{ik}	set of positions that can be occupied by unit k of product i: $H_{ik} = \{h—E_{ik} \leqslant h \leqslant L_{ik}\}$ ($i = 1,\dots,n$; $k = 1,\dots,d_i$)

Variables:

$y_{ikh} \in \{0,1\}$ 1 if and only if unit k of product i is placed in position h ($i = 1,\dots,n$; $k = 1,\dots,d_i$; $h \in H_{ik}$)

$\delta_{ik}^j \in \{0,1\}$ 1 if and only if the distance between units k and $k + 1$ of product i is equal to j ($\forall$ $i \in G1$; $k = 1,\dots,d_i$; $j = 1,\dots,UB_i$)

Model MILP-0:

$$[MIN]RTV = \sum_{\forall i \in G1,k,j} j^2 \cdot \delta_{ik}^j - \sum_{i \in G1} d_i \cdot \bar{t}_i^2 \tag{1}$$

$$\sum_{\forall (i,k)|h \in H_{ik}} y_{ikh} = 1 \quad (h = 1, \dots, D) \tag{2}$$

$$\sum_{h \in H_{ik}} y_{ikh} = 1 \quad (i = 1, \dots, n; k = 1, \dots, d_i) \tag{3}$$

$$\sum_{h \in H_{i,k+1}} h \cdot y_{i,k+1,h} - \sum_{h \in H_{ik}} h \cdot y_{ikh} = \delta_{ik}^1 + \dots + j \cdot \delta_{ik}^j + \dots + UB_i \cdot \delta_{ik}^{UB_i} \quad (\forall i \in G1; k = 1, \dots, d_i - 1) \tag{4}$$

$$D - \sum_{h \in H_{id_i}} h \cdot y_{i,d_i,h} + \sum_{h \in H_{i1}} h \cdot y_{i1h} = \delta_{id_i}^1 + \dots + j \cdot \delta_{id_i}^j + \dots + UB_i \cdot \delta_{id_i}^{UB_i} \quad (\forall i \in G1) \tag{5}$$

$$\sum_{j=1}^{UB_i} \delta_{ik}^j = 1 \quad (\forall i \in G1; k = 1, \dots, d_i) \tag{6}$$