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Exact performance analytical model for spectrum allocation in flexible grid optical networks

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ABSTRACT

Dynamic flexible grid optical networks have gained much attention because of the advantages of high spectrum efficiency and flexibility, while the performance analysis will be more complex compared with fixed grid optical networks. An analytical Markov model is first presented in the paper, which can exactly describe the stochastic characteristics of the spectrum allocation in flexible grid optical networks considering both random-fit and first-fit resource assignment policies. We focus on the effect of spectrum contiguous constraint which has not been systematically studied in respect of mathematical modeling, and three major properties of the model are presented and analyzed. The model can expose key performance features and act as the foundation of modeling the Routing and Spectrum Assignment (RSA) problem with diverse topologies. Two heuristic algorithms are also proposed to make it more tractable. Finally, several key parameters, such as blocking probability, resource utilization rate and fragmentation rate are presented and computed, and the corresponding Monte Carlo simulation results match closely with analytical results, which prove the correctness of this mathematical model.

1. Introduction

1.1. Backgrounds and motivations

Flexible Grid Optical Networks (FGON) have recently attracted much interest of industries and research institutes, for their ability to allocate spectrum and capacity dynamically to meet the demand of incoming traffics. FGON has been viewed as the evolutionary transmission and switching architecture from the mature Wavelength-Division Multiplexing (WDM) Optical Networks, for the reason that in WDM Networks bit rates more than 100 Gb/s will not easily fit into the 50 GHz International Telecommunication Union (ITU) wavelength grid. This results in handicaps of satisfying the skyrocketing bandwidth demands especially from large scale data centers, high-definition video content providers, and concurrent computation [1]. On the opposite side, to establish a lightpath in a flexible grid optical network, each bandwidth variable spectrum cross connect (BV-WSS) in the potential route allocates sufficient spectrum to build an appropriately sized end-to-end optical channel [2]. Due to those competitive virtues, the ITU-T and IETF have started to draw up the relevant standard documents, and define finer channel spacing and the new set of DWDM central frequencies to meet the demand of flexible granularity [3]. Besides, Multi-carrier solutions such as coherent optical orthogonal frequency-division multiplexing (CO-OFDM), as well as Nyquist-WDM have been proposed as possible transponder implementations for FGON [1].

In FGON, the usable fiber spectrum resources are practically quantized into contiguous frequency units (FU) with an appropriate width, e.g., 12.5 GHz, to simplify the network design and modeling. Those are the building blocks of the resource range occupied by optical channels, enabling resource allocation with relatively arbitrary granularity. The evolution of resource units from wavelengths to flexible frequency units leads to fundamental changes of the spectrum allocation manner. Those changes bring a new form of constraints, called *Spectrum Contiguity Constraint* (SCC) in this paper, to spectrum allocation systems, which can be expressed as that the frequency units on a link occupied by one optical channel must be contiguous. This constraint is different from the traditional *Wavelength (Spectrum) Continuity Constraint* in both WDM networks and FGONs, which means a connection can only be established if the same wavelength (spectrum) is available on all the links along the designated path [4].

Blocking probability and resource utilization rate are two common metrics of performance in WDM networks [5], which can be used to dimension network and link capacities while taking into account the dynamics of lightpath requests. The former is the probability that a call cannot be accepted, while the latter is the

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percentage of the utilized resources out of the total resources. These two metrics are calculated typically using queuing models or more general Markov models [12], which can characterize the dynamic features of a stochastic system.

The SCC introduces load correlation between frequency units in a spectrum allocation system (which can also be seen as a single-link FGON). The queuing model or the Markov model with independence assumptions cannot be applicable to characterizing the exact states of the system. For instance, SCC keeps the system from being modeled by the number of idle FUs which cannot point out the correlation between FUs. This results in much more difficulty in evaluating the performance, such as blocking probability and resource utilization rate, which can be treated as criteria of qualities and effectiveness of a spectrum allocation system and the corresponding allocating policies. Furthermore, there is a special metric called *fragmentation rate* in FGON, which represents the orderliness of spectrum and is defined in this paper. So far, limited work has been done to address this problem. [6] propose a birth–death model to analyze the performance of time-varying traffics in flexible optical networks, but the RSA problem is transformed into a static one. [7] analyze the blocking probability under special network status with certain fragmentation and utilization.

1.2. Previous models and their scope

The study of the performance of a single-link FGON is the prerequisite of the corresponding network with an arbitrary topology. A single-link FGON or spectrum allocation system can be modeled as a special kind of multi-server queuing system. There have been several practical models that can be used for various queuing systems. The simplest one is the Erlang loss model, i.e. the $M/M/C/C$ model in Kendall notation, which has been widely used in evaluating the performance of circuit-switched systems [8–10]. This model depicts the state of system as the number of calls in the system with Poisson arrival and holding time. Under the condition that there are more than one classes of calls who will request distinct number of FUs of the system, the multi-dimensional Erlang loss model can be used, whose states require more than one variables, and each variable represents the number of corresponding calls in the system [11–13]. Another variant is the batched arrival/departure Erlang loss model, in which call arrivals/departures are modeled as a batched Poisson process with an arbitrary batched size distribution [14]. [15] proposes the model $M^X/M/C/C$ for measuring the OFDM subcarrier allocation, and a more adequate model is proposed by Pla in [16]. Most of the papers [9–18] adopt Erlangs models or derivative ones for their specific and particular contexts.

There is a fundamental postulation in Erlang's models that the sequence of servers in the system is ignored, i.e. a call can occupy several arbitrarily chosen idle servers without any constraint. As a consequence, Erlang's models are no longer adequate to exactly describing FGON due to the effect of contiguity constraint, because under this constraint it must be considered that which servers will be occupied when a call arrives.

In addition, FGON could not be modeled under the equivalence assumption of FUs in a link. When the random-fit policy is applied in the wavelength assignment procedure in WDM networks, wavelengths in a link can be seen as they are mutually independent and are equally likely to be available. Thus, some special properties such as the inclusion–exclusion principle hold [8]. But in FGON, even applying the random-fit policy, each FU in a link is affected by the adjacent FUs and cannot be treated equally due to the contiguity constraint. This can be simply explained that the lowest numbered FU and the highest numbered FU just have one side contiguous FUs, so that they are distinct from other FUs, and it is easy to deduce that each two FUs are distinct if they have different distance to the margin of resource set. In the situation that the first-fit policy is applied, there exists heterogeneity between FUs in both WDM networks and FGON. [19–21] propose versions of overflow traffic models for the first-fit policy in WDM networks.

Furthermore, It should be noted that if all the potential calls require the same amount of FUs, the blocking model for FGON degenerates to a conventional WDM model, for in such a situation the FUs required by a call can be bundled and seen as a wavelength in the corresponding WDM networks. Hence, multi-class models should be given in the papers discussing the modeling of FGON. The meaning of *class* should be clarified. In this paper, calls are *classified* by the number of FUs required per call. There are fewer papers addressing multi-class models than single-class ones due to the complex nature of the problem. Multi-class circuit-switching networks is analyzed in [22–24], and the approach of [23] is fully utilized by [25], addressing a multi-class WDM service with wavelength continuity constraint employing the knapsack approximation.

1.3. Our contributions

We can conclude that the model of FGON should have the following attributes.

- There is load correlation between FUs in a link due to the spectrum contiguity constraint. Assumptions ignoring the sequence of FUs cannot be accepted.
- There is load heterogeneity between FUs in a link for any spectrum assignment policy due to the spectrum contiguity constraint. Assumptions that each FU in a link are equivalent cannot be accepted.
- There is more than one class of underlying calls, each of whom in different classes requires distinct amount of FUs. It must be a multi-class model.

In this paper, our major contribution is an exact performance analytical model of the spectrum allocation system and flexible grid optical links (abbreviated as *the EPA model* in this paper), considering all of the above attributes. We also present detailed analysis of its mathematical nature, stochastic properties and efficient computing methods. The EPA model is a good breakthrough point to start with in the challenging task of performance analysis of FGON. Under this model, we propose formulae to calculate the blocking probability, as well as the resource utilization rate and the fragmentation rate. The corresponding Monte Carlo simulation results are presented and show high accuracy of our analytical model. Part of our work has been introduced in [27]. This is the first time, to the best of our knowledge, to exactly analyze the performance of flexible grid optical networks in detail. It is awaiting improvements that the EPA model is computation intensive with the complexity growing exponentially with the number of FUs in a link. However, the major value of the EPA model is that it characterizes the behavior of the spectrum allocation without any approximation and can be treated as the evaluation criterion of other models.

Our work is driven by the ineffectiveness of conventional models due to the spectrum contiguity constraint. On the contrary, the multi-link model is out of the scope of this paper because there have been plenty of multi-link modeling methods capable of being used for FGON without or with slight modifications, such as the reduced load approximation method [9] or the path decomposition method [26]. Besides, the guard bands [1] dividing adjacent optical