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Performance evaluation of random linear network coding using a Vandermonde matrix

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ABSTRACT

This paper discusses random linear network coding with and without the use of a Vandermonde matrix to obtain the coding coefficients. Performance comparisons of such random linear network coded networks with networks employing traditional store and forward technique are also provided. It is shown that random linear network coding using a Vandermonde matrix can improve the network utilization factor by reducing the overhead compared to random linear coding that does not use a Vandermonde matrix. Our numerical results show that random linear network coding with a Vandermonde matrix provides a considerable improvement in throughput and delay when compared to a network employing a traditional store and forward strategy. An inherent feature of random linear network coding which makes it possible to employ simple encryption techniques is also discussed.

1. Introduction

Network coding, in simple terms, is a technique that relies on combining independent packets at intermediate nodes and then forwarding them such that they could be recovered at their respective destinations. The technique is considered by several researchers as an innovative extension of the traditional store and forward paradigm and can effectively reduce packet density in communication networks. It increases the utilization of networks by reducing the expected number of transmissions and can be applied at symbol level in the physical layer, byte level in the MAC layer or at the packet level in a layer above the MAC layer. Network coding becomes robust in situations where the knowledge of topology is difficult to obtain or in the presence of link failures as in a Wireless Sensor Network (WSN) or in ad hoc networks. Due to its potential advantages, network coding has gained popularity in various wired and wireless applications and has been proposed for efficient data dissemination over WSN [1,2] and internet multimedia streaming [3,4].

A few other relevant and interesting studies on network coding are as follows. In [5], diversity network codes over finite fields based on linear network coding were applied for multiuser cooperative communication. For a network consisting of M users, it has been shown in [5] that a diversity of order $(2M - 1)$ can be achieved. Xiao et al. in [6,7] proposed a binary deterministic rate-less code for combination networks based on a sparse transfer matrix. Encoding is done using cyclic shifts and XOR's. They show that the encoding complexity is linear. However, it can be used for only small set of packets.

Recently, Random Linear Network Coding (RLNC) [8,9] has received considerable attention from the research community as a technique to counter adverse effects as delays, fading, and erasures of wireless communication links. RLNC is attractive from the viewpoint of higher throughput and better utilization of network resources and is also one of the most suitable algorithms when network

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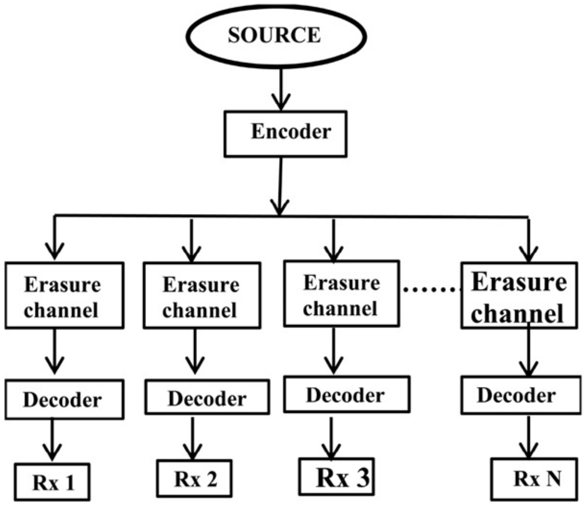

Fig. 1. System model.

topology is unknown, or there are any link failures, or in any other error prone environment.

To outline briefly the idea behind RLNC, consider a source that has to transmit M packets to a set of N receivers through N independent erasure channels as in Fig. 1. In RLNC, the encoder at the source linearly combines the M packets and sends this combination as a single coded packet in the jth time slot. Eq. (1) below represents the encoder operation for RLNC.

$$C_j = \sum_{i=1}^{M} \beta_{ij} P_i. \quad (1)$$

Here β_{ij} is the coding coefficient of the ith packet P_i, in the jth time slot. Usually, β_{ij} is selected randomly from a Galois Field (GF) of appropriate size. At the receiver side, with the knowledge of the coefficients β_{ij}, the individual packets could be decoded from any M coded packets received, provided the M coefficients used to combine the packets to form these M coded packets could be stacked to form an $M \times M$ invertible matrix. A major drawback here is that, since the coefficients are selected randomly, the invertibility condition mentioned above cannot be guaranteed. Secondly, the transmission of the M coefficients β_{ij} ($i = 1$ to M) to the receivers to facilitate recovery of packets adds to network overhead. Needless to say, both these drawbacks impact the throughput negatively. More details on RLNC over finite fields and its mathematical structure can be found in [10–12], and references therein.

To overcome such drawbacks of RLNC while retaining its advantages, an alternative approach to RLNC is proposed wherein the coded packet is generated with the coding coefficients as elements of the rows of a Vandermonde matrix [13–16]; we identify such RLNC which is based on Vandermonde matrix as RLNCV. It is well known that with the number of columns as M, any M rows of a Vandermonde matrix will be linearly independent. This ensures the invertibility condition mentioned above.

RLNCV, like RLNC, is suitable for almost any network topology as it only differs in the way the coding vectors are generated and notably, generation of coding vectors is independent of network topology. An important significance of RLNCV considering a Vandermonde matrix for the generation of coding vectors is that there is no need to send all of the elements of the coding vector since an entire row of a Vandermonde matrix can be constructed using a single non-zero field element. This effectively compresses the network header to just one symbol as compared to M symbols. It is worth mentioning here that to the best of our knowledge, no MDS code can compress the network header to such extent as achieved by using a Vandermonde matrix for generating the coding vectors.

Though usage of a Vandermonde matrix for network coding has been proposed earlier by other researchers, to the best of our knowledge, published literature does not have a detailed and comprehensive performance analysis of RLNC and RLNCV with appropriate comparisons to a traditional Store and Forward technique (SF). In this paper, we aim to bridge this important gap to a large extent. Our analysis and results quantify the performance of RLNCV as well as RLNC over finite fields. Our results also show rigorously the significant enhancements in network performance parameters such as throughput and delay, achieved by RLNC and RLNCV compared to SF. Further, we also show analytically the enhancement in network utilization factor achieved by RLNCV over RLNC.

In the remaining part of the paper, in Section 2 we compare RLNCV with RLNC with regards to the utilization factor and choice of coding coefficients. Later, in Section 3, we discuss some numerical results which bring out the performance of RLNCV and RLNC and comparisons of their performance with that of SF. Section 4 concludes the paper.

2. RLNCV and RLNC

Consider a system as earlier and as depicted in Fig. 1. With the traditional RLNC replaced by RLNCV, Eq. (1) which governs the packet encoding, gets altered to Eq. (2) as given below.

$$C_j = \sum_{i=1}^{M} (\beta_j)^{(i-1)} P_i. \quad (2)$$