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Belief functions on general intuitionistic fuzzy information systems

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ABSTRACT

This paper studies probability problems of intuitionistic fuzzy sets and the belief structure of general intuitionistic fuzzy information systems. First, for some special intuitionistic fuzzy sets, a probability is defined by using the integral operations on the level sets, and its properties are discussed. Then using this probability, a mass function of an intuitionistic fuzzy information system is constructed. A novel pair of belief and plausibility functions are defined by employing the $(\mathcal{I},\mathcal{T})$ intuitionistic fuzzy lower and upper approximation operators. Finally, an example in decision analysis is employed to illustrate the application of the new belief and plausibility functions.

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1. Introduction

The theory of rough sets, proposed by Pawlak in 1981 [24], is a new mathematical tool for data reasoning. As a mathematical method to deal with insufficient and incomplete data, it is a set-theory-based technique to handle data with granular structures by using two sets called the rough lower approximation and the rough upper approximation to approximate an object. By using this method, knowledge hidden in information systems may be revealed and expressed in the form of decision rules. The main idea of rough set theory is using the existing knowledge to approximate uncertain concepts and phenomena [25]. With more than 30 years development, rough set theory has been found to have very successful applications in the fields of artificial intelligence such as expert systems, machine learning, pattern recognition, decision analysis, process control and knowledge discovery in databases. The classical definition of a *Pawlak* rough set is with reference to an equivalence relation. From both theoretical and practical viewpoints, the classical equivalence relation is a very stringent condition that may limit applications of rough sets. Many authors have generalized the notion of rough set approximations by using nonequivalence binary relations [5,21,31,32,41,42], and the non-equivalence relation-based rough set models have been used in reasoning and knowledge acquisition with data sets presented as incomplete information tables. Recently, many authors studied rough approximations in fuzzy environment, and the results are called rough fuzzy sets and fuzzy rough sets [12,23,26,36,43,46,47]. These models have been employed to handle fuzzy and quantitative data. As a more general case of fuzzy sets, the concept of Atanassov intuitionistic fuzzy sets (A-IFSs, for short), which was originated by

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Atanassov [1], has played a useful role in the research of uncertainty theories. A fuzzy set only gives a membership degree to describe an element belonging to a set, while an Atanassov intuitionistic fuzzy (A-IF, for short) set gives both a membership degree and a nonmembership degree. Thus, an A-IF set is more objective than a fuzzy set to describe the vagueness of data or information. Concepts and operations of fuzzy sets are generalized into A-IF sets [2,3,6,8,19,20,22,48]. More recently, rough set approximations were introduced into A-IFSs by many authors. They respectively proposed the concept of A-IF rough sets in which the lower and upper approximations are both A-IFSs [9,27,29]. Then the definition and properties of approximations of A-IFSs are investigated w.r.t. an arbitrary A-IF binary relation in which the universe of discourse may be infinite [37,49].

Another important method used to deal with uncertainty in information systems is the Dempster–Shafer theory of evidence (evidence theory, for short). The evidence theory is a method developed to model and manipulate uncertain, imprecise, incomplete, and even vague information. It was originated from Dempster's concept of lower and upper probabilities [11], and extended by Shafer [30] as a theory. The basic representational structure in this theory is a belief structure, which consists of a family of subsets, called focal elements, with associated individual positive weights summing to 1. The fundamental numeric measures derived from the belief structure are a dual pair of belief and plausibility functions. Since its inception, evidential reasoning has emerged as a powerful methodology for pattern recognition, image analysis, diagnosis, knowledge discovery, information fusion, and decision making [35,38,44]. In [14], Dymova and Sevastjanov defined the value of belief function of an A-IFS A with respect to an element x by using the membership degree, and defined the value of plausibility function by 1 minus the non-membership degree.

There are strong connections between rough set theory and the evidence theory. It has been demonstrated that certain belief structures are associated with certain rough approximation spaces such that the different dual pairs of lower and upper approximation operators induced by the rough approximation spaces may be used to interpret the corresponding dual pairs of belief and plausibility functions induced by the belief structures [7,28,39–41]. In order to promote the application scope of A-IF information systems, we want to discuss decision analysis and information fusion in A-IF information systems by using the evidence theory. However, the existing results of the evidence theory handle mostly crisp or fuzzy information, and it cannot be directly extended to A-IF information systems (since A-IF sets give a membership degree and a nonmembership degree). Therefore, in order to better make the rules and information fusion, we want that both of the values of the belief function and the plausibility function are real numbers (since two IF numbers cannot compare the size, sometimes). As the evidence theory is closely related to probability theory, we need to define a convenient and calculable probability for an A-IFS, which meets the above requirements. Moreover, we can construct belief structure of A-IF systems by the probability of A-IF sets and the lower and upper A-IF approximation operators. In [4], Beloslav Riečan defined a descriptive concept of probability on A-IF sets, where the probability of an A-IF set is an interval value. In [15,16], Tadeusz Gerstenkorn and Jacek Mańko gave two new definitions of the A-IF probability: the first probability of an A-IF set is a real number in $[0,1]$ using the integral operation, and the second probability of an A-IF set is also an A-IF set based on the level sets. In [17], Tadeusz Gerstenkorn and Jacek Mańko defined a probability of A-IF events, which is defined by the membership degree and half of the hesitancy margin of every element, where the probability of an A-IF set is a real number. Since an A-IF set also can be viewed as an A-IF event, we tend to define the probability of an A-IF set be a real number in $[0,1]$. In this paper, we propose a new method to compute the probability of an A-IF set using the double integral operation on the level sets, which is a real number in $[0,1]$. We then study the evidence theory of A-IF systems using our proposed probability of an A-IF set and the A-IF $(\mathcal{I},\mathcal{T})$ rough approximation operators.

The paper is organized as follows: Section 2 reviews the definitions and properties of A-IF sets and A-IF $(\mathcal{I},\mathcal{T})$ approximation spaces. In Section 3, a novel probability of an A-IF set by using the (α,β) level set is defined and some basic properties are discussed. Section 4 studies the belief structure and the belief and plausibility functions based on our proposed probability, an example in decision analysis is presented to illustrate the application of our method. We then conclude the paper with a summary and outlook for further research in Section 5.

2. Basic concepts

2.1. Intuitionistic fuzzy $(\mathcal{I},\mathcal{T})$ approximation space

In this subsection, we first recall a special lattice on $[0,1]\times[0,1]$ (where $[0,1]$ is the unit interval) and its logical operations originated by Cornelis et al. [10]. And then we review some basic concepts about the A-IF $(\mathcal{I},\mathcal{T})$ approximation operators [10,49]. These concepts are under the A-IF environment and may be viewed as generalizations of the fuzzy logical connectives on $([0,1],\leqslant)$.

Let U be a nonempty set called the universe of discourse. The class of all subsets (respectively, fuzzy subsets) of U will be denoted by $\mathcal{P}(U)$ (respectively, by $\mathcal{F}(U)$). For any $A\in\mathcal{F}(U)$, the α-level and strong α-level of A will be denoted by $(A)_\alpha$ and $(A)_{\alpha+}$, respectively, that is, $(A)_\alpha=\{x\in U: A(x)\geqslant\alpha\}$ and $(A)_{\alpha+}=\{x\in U: A(x)>\alpha\}$, where $\alpha\in[0,1]$, the unit interval, $(A)_0=U$, and $(A)_{1+}=\emptyset$. And $(A\cup B)(x)=\max\{A(x),B(x)\},(A\cap B)(x)=\min\{A(x),B(x)\},\sim A(x)=1-A(x),\forall A,B\in\mathcal{F}(U)$. For $\alpha\in[0,1]$, $\widehat{\alpha}$ is the constant fuzzy set: $\widehat{\alpha}(x)=\alpha$, for all $x\in U$.

Denote $L^*=\{(x_1,x_2)\in[0,1]\times[0,1]: x_1+x_2\le 1\}$. We define a relation $\le_{L^*}$ on L^* as follows: $\forall(x_1,x_2),(y_1,y_2)\in L^*$,

$$(x_1,x_2)\le_{L^*}(y_1,y_2)\Longleftrightarrow x_1\leqslant y_1 \text{ and } x_2\geqslant y_2.$$

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