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A modified ant colony optimization algorithm for dynamic topology optimization

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ABSTRACT

A modified ant colony optimization (MACO) algorithm implementing a new definition of pheromone and a new cooperation mechanism between ants is presented in this paper. The sensitivity of structural response to the presence of each element included in the finite element (FE) model is evaluated. The study aims to improve the suitability and computational efficiency of the ant colony optimization algorithm in dynamic topology optimization problems. The natural frequencies of the structure must be maximized yet satisfying a constraint on the final volume. Optimization results obtained in three test cases indicate that MACO is more efficient and robust than standard ACO in solving dynamic topology optimization problems.

1. Introduction

The ant colony optimization (ACO) algorithm is a metaheuristic search method for global optimization. ACO was initially proposed by Dorigo [1] to find the optimal path in a graph. ACO mimics the behavior of ants seeking a path between their colony and a food source. This optimization technique was successfully applied to many engineering problems including structural optimization [2–4].

Topology optimization of structures underwent tremendous development after the introduction of the homogenization method [5]. In addition, ad hoc optimization algorithms such as evolutionary structural optimization (ESO) [6,7], performance-based optimization (PBO) [8,9], and level set theory [10,11] were successfully applied to topology optimization problems.

The homogenization method divides the design space in an infinite number of microscale holes and optimal topology is obtained by solving a material distribution problem. However, it may yield an undesirable structure with infinitesimal pores in the materials that make the structure not realizable practically [12]. The rationale of ESO and PBO is to remove gradually from the structure, the "inefficient" elements hosting very low stress that hence do not contribute much to the overall response of the structure: such an operation is governed by the sensitivity number. Convergence behavior of PBO depends on the performance of the structure. Bidirectional ESO (BESO) [13] was developed to improve the capability of adding or removing elements. However, the computational efficiency of BESO depends on the previous positions of the elements as well as on the area/volume of the element predefined by the meshing operation.

Topology optimization can be performed with metaheuristic algorithms that mimic natural phenomena and physical processes. Applications of genetic algorithms (GA) [14] and simulated annealing (SA) [15] to topology optimization problems were reviewed in the paper written by Luh and Lin [16]. Also, ant colony optimization was utilized in topology optimization problems [16,17]. For example, Luh and Lin [16] used the element transition rule instead of node transition rule and connectivity analysis, pheromone updating rule and multiple-colony memories. Kaveh et al. [17] developed a topology optimization technique to find the stiffest structure with a certain amount of material; the search criterion was based on the contribution of each element to strain energy. It was found that ACO can handle the topology optimization problem as an on-off discrete optimization. However, there are not many other examples of applying ACO to topology optimization problems documented in literature.

Although dynamic topology optimization with respect to natural frequencies is of fundamental importance in aerospace and automotive engineering, the number of papers published on this subject is limited in comparison with the available literature on static problems. For example, the homogenization method [18,19] and modified SIMP (Solid Isotropic Microstructure with Penalization) with a discontinuous function [20–22] were successfully used to solve eigenvalue problems in topology design of vibrating structures.

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In this study, a modified ant colony optimization (MACO) algorithm is developed in order to improve computational efficiency and suitability of ACO in topology optimization problems dealing with natural frequencies. An important improvement with respect to classical ACO is the definition of a continuous variable, the element contribution significance (ECS), which serves to evaluate the effective contribution to structural response deriving from the presence of each element. A mesh independent filtering scheme [23] is adopted to prevent the formation of checkerboard patterns in the optimization process. Optimal designs are compared with those obtained with standard ACO and soft-kill BESO in order to assess the applicability and the efficiency of the proposed MACO algorithm in dynamic problems.

2. Topology optimization for dynamic problems

2.1. Formulation of dynamic topology optimization problems

Excessive vibration due to resonance occurs when the frequency of the dynamic excitation is close to a natural frequency of the structure. Therefore, it is necessary to restrict the fundamental frequency or several of the lower frequencies of the structure to a prescribed range in order to avoid severe vibration.

In the finite element method, the dynamic behavior of a structure is represented by the following general eigenvalue problem [24];

$$([K] - \omega_i^2[M])\{u_i\} = \{0\} \tag{1}$$

where $[K]$ is the global stiffness matrix, $[M]$ is the global mass matrix, ω_i is the ith natural frequency and $\{u_i\}$ is the eigenvector corresponding to ω_i. The natural frequency ω_i and the corresponding eigenvector $\{u_i\}$ are related by the Rayleigh quotient as follows;

$$\omega_i^2 = \frac{\{u_i\}^T[K]\{u_i\}}{\{u_i\}^T[M]\{u_i\}} \tag{2}$$

Dynamic topology optimization problems where the objective is to maximize the ith natural frequency of a continuum structures are considered in this research. For a solid-void design, the optimization problem can be stated as [24];

$$\text{Maximize}: \omega_i$$

$$\text{Subject to}: V^* - \sum_{i=1}^{N} V_i x_i = 0 \tag{3}$$

$$x_i = \begin{cases} 1 & \text{if element is solid} \\ x_{\min} & \text{if element is void} \end{cases}$$

where V_i is the volume of an individual element, and V^* is the prescribed volume. N is the total number of elements in the structure. The binary design variable x_i denotes the density of the ith element; $x_{\min}$ is set equal to a sufficiently small value to denote a void element.

2.2. Material interpolation scheme

To obtain the gradient information of the design variable, it is necessary to interpolate the material between $x_{\min}$ and 1. A popular material interpolation scheme is the so-called power-law penalization model (the SIMP model). For the solid–void design, the material density and Young's modulus are functions of the design variable x_i as [24];

$$\rho(x_i) = x_i \rho^1 \tag{4}$$

$$E(x_i) = x_i^p E^1 (0 < x_{\min} \leqslant x_i \leqslant 1) \tag{5}$$

where ρ^1 and E^1 are the density and Young's modulus of solid material, respectively. p is the penalty factor, usually used as 3.

However, the SIMP model described above cannot be directly applied to dynamic topology optimization problems where the objective is to maximize target frequencies. This is because the very high ratio between penalization on mass and stiffness for small values of x_i may result in the presence of localized modes in the low-density [21].

One idea to avoid this problem is to keep the ratio between mass and stiffness constant when $x_i = x_{\min}$ by requiring that

$$\rho(x_{\min}) = x_{\min} \rho^1 \tag{6}$$

$$E(x_{\min}) = x_{\min} E^1 \tag{7}$$

Therefore, an alternative material interpolation scheme can be expressed by Huang et al. [24];

$$\rho(x_i) = x_i \rho^1 \tag{8}$$

$$E(x_i) = \left[\frac{x_{\min} - x_{\min}^p}{1 - x_{\min}^p}(1 - x_i^p) + x_i^p\right] E^1 \quad (0 < x_{\min} \leqslant x_i \leqslant 1) \tag{9}$$

The change in natural frequency $\Delta\omega_i$ caused by the removal of the generic ith element from the structure indicates the extent to which the global structural response is sensitive to that element. Therefore, the change in natural frequency can be taken as the quantity of pheromone in the ant colony optimization process. The change in natural frequency $\Delta\omega_i$ can be defined as follows [25];

$$\Delta\omega_i = \begin{cases} \dfrac{1}{2\omega_i} u_i^T \left(K_i - \dfrac{\omega_i^2}{p} M_i\right) u_i, & (x = 1) \\ -\dfrac{\omega_i}{2p} u_i^T M_i u_i, & (x = x_{\min}) \end{cases} \tag{10}$$

where M_i is the mass matrix of each element, K_i is the stiffness matrix of each element, and u_i is the element eigenvector which are related to the removed each element.

3. Ant colony optimization formulations for dynamic topology problems

3.1. Standard ACO applied to dynamic topology optimization

The ant colony optimization algorithm (ACO) mimics the behavior of real ant colonies. Ants can find the shortest path from a food source to their nest by exploiting a chemical substance called pheromone. Each insect deposits a trail of pheromone on the ground. This trail adds to the previously deposited pheromone trails. The other ants will be more likely to go through a route if this hosts a higher concentration of pheromone.

ACO was successfully utilized to solve the travelling salesman problem (TSP), a classical combinatorial optimization problem. Kaveh et al. [17] developed a standard ACO formulation for static topology optimization problems in order to design the stiffest structure with the constraint of disposing of a certain amount of material; the search criterion was based on strain energy. Following the logical scheme adopted to solve the TSP, standard ACO will be applied in this study to dynamic topology problems. Elements of discretized design domain correspond to distances covered by the salesman while elements forming the structure under the given volume constraint correspond to the cities.

The contribution of each element i to the cost function of the optimization problem resembles the pheromone trail deposited on a segment of a route, it is here denoted by $\tau_i(t)$. The parameter t represents the time of deployment of ants which is equivalent to iteration cycles. Following the approach adopted for TSP [1], and ignoring the effect of the local heuristic values, the ant decision index $a_i(t)$ can be written as [17];