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Finding optimal strategies in a multi-period multi-leader–follower Stackelberg game using an evolutionary algorithm

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ABSTRACT

Stackelberg games are a classic example of bilevel optimization problems, which are often encountered in game theory and economics. These are complex problems with a hierarchical structure, where one optimization task is nested within the other. Despite a number of studies on handling bilevel optimization problems, these problems still remain a challenging territory, and existing methodologies are able to handle only simple problems with few variables under assumptions of continuity and differentiability. In this paper, we consider a special case of a multi-period multi-leader–follower Stackelberg competition model with non-linear cost and demand functions and discrete production variables. The model has potential applications, for instance in aircraft manufacturing industry, which is an oligopoly where a few giant firms enjoy a tremendous commitment power over the other smaller players. We solve cases with different number of leaders and followers, and show how the entrance or exit of a player affects the profits of the other players. In the presence of various model complexities, we use a computationally intensive nested evolutionary strategy to find an optimal solution for the model. The strategy is evaluated on a test-suite of bilevel problems, and it has been shown that the method is successful in handling difficult bilevel problems.

1. Introduction

The relevance of bilevel optimization problems has been amply recognized by researchers and practitioners alike [1]. The key difference between bilevel programming problems and other optimization problems is their nested structure. A bilevel program is commonly defined as an optimization problem, which contains another optimization task within the constraints of the outer problem. The outer optimization problem is generally termed as the upper level problem, and the constraining optimization task is denominated as the lower level problem. Such a nested structure means that a solution to the upper level problem is considered feasible only if it is an optimal solution to the lower level problem. Because of this requirement, bilevel optimization problems can quickly become very difficult to solve.

Bilevel optimization problems often appear as leader–follower problems in the fields of game theory and economics. When formulating a leader–follower scenario as a bilevel programming problem, the leader's optimization task is modeled at the upper level, constrained by the follower's optimization task at the lower level. The leader has the ability to move first, and is assumed to possess all necessary information about the follower's possible reactions to the actions taken by the leader. The follower, on the other hand, observes the leader's actions before reacting optimally to them. By solving a Stackelberg competition model, the leader can forecast the follower's reactions and determine his own optimal actions.

In this paper, we model a particular kind of oligopolistic market involving multiple leaders and followers interacting over multiple time periods. Such a scenario can be analyzed as a special case of a multi-period multi-leader–follower Stackelberg game. This problem builds on the original definition of duopolistic competition proposed by Stackelberg [2] and its various extensions [3–6]. The multi-period market model presented in this paper allows us to incorporate relevant dynamics and several practical aspects, such as investment and marketing effects, in a flexible manner. As an extension to the prior work, we also assume the production of all players in this market to be discrete variables while allowing the investment and marketing decisions to be continuous. This feature makes the model framework applicable to markets where the products are valuable and are not well approximated by continuous variables, such as in the aircraft manufacturing market. Further realism is added to the problem in the form of non-linear demand and cost functions, and constraints on investment and marketing budget for both the leaders and the followers. The effects of investment and marketing are considered to be

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cumulative leading to interactions between the time-periods. Evolutionary algorithms have been used in the past in the domain of game theory with players making simultaneous moves (Nash Equilibria), players making asynchronous moves, or a mix of the two situations. For instance, Lung and Dumitrescu [7] consider a multi-player game and propose a domination concept to detect the Nash-equilibria using an evolutionary multi-objective algorithm. Koh [8] proposes an evolutionary algorithm for handling a similar model with multiple leaders and followers as considered in this paper. However, Koh assumes that it is possible to write the first-order conditions for the lower level problem, which is not possible in the problem considered in this paper. Classical techniques like branch-and-bound have also been applied to handle bilevel problems involving multiple players. Lu et al. [9] use a branch-and-bound technique to handle linear Stackelberg problems with a single leader and multiple followers. Zhang et al. [10] study a similar problem with a single leader and multiple followers, but in this study the functions involve fuzzy coefficients and multiple objectives at both levels.

Many studies have been conducted in the field of bilevel programming [11–14] and on its practical applications for solving various problems [1]. Aspects of bilevel programming have also been reviewed in detail by Colson et al. [11] and Vicente and Calamai [12]. The problems encountered in the literature are commonly solved with the help of approximate solution methodologies [15–17]. These techniques work with numerous simplifying assumptions, and most of them are inapplicable to problems with higher levels of complexity. The classical methods widely employed by researchers and practitioners to solve these problems include the Karush–Kuhn–Tucker approach [15,18], Branch-and-bound techniques [19] and the use of penalty functions [20]. However, the recent technological advances and increased computing power have made heuristic approaches, such as evolutionary algorithms, more popular for solving complex optimization problems. Evolutionary algorithms have also been applied to bilevel programming problems [5,21–23,14]. Given the encouraging results obtained with such techniques, we have also opted to employ a nested evolutionary algorithm to solve the multi-leader-flower problem. The choice is well motivated by the complexities due to non-linearity and discreteness that are inherent in our model for oligopolistic markets.

The remainder of the paper is structured as follows. Section 2 presents a general formulation of a multi-period multi-leader–follower Stackelberg competition model. Section 3 outlines a more concrete model with a discussion about its possible application to the aircraft manufacturing market. Section 4 contains a description of the procedure followed by the nested bilevel evolutionary algorithm to arrive at the optimal solution, and then in Section 5 we evaluate the proposed nested bilevel evolutionary algorithm. Section 6 presents the results on the multi-period multi-leader–follower Stackelberg optimization problem. Section 7 provides a convergence analysis on a simplistic version of the Stackelberg competition model. Lastly, the conclusions and plans for future work are summarized in Section 8.

2. Generalized Stackelberg competition model

Three main models – the Bertrand, Cournot, and Stackelberg competition models – are extensively applied in economics when modeling multi-firm competition [4,6,24]. The first two models are generally used when the competing firms have an approximately equal amount of market power, and make production and pricing decisions simultaneously. However, in an oligopoly, where some of the competing firms have more market power than others, a Stackelberg model is considered more suitable. Such markets are much more common in practice than strict oligopolies and deserve more attention [3,6]. In this section, we outline some of the notations used throughout the rest of this paper and present a generalized formulation of a Stackelberg competition model with multiple leaders and followers.

In the context of the multi-leader–follower problem framework, a strategy is defined as a sequence of decisions made by a player throughout the duration of the game. A strategy for a leader i can be stated as $s_{l,i}=(s_{l,i}^t)_{t=1}^T \in S_{l,i}$, where $s_{l,i}^t$ is a particular decision made by the leader at time t, and $S_{l,i}$ is the set of all alternative strategies available to leader i. Similarly, we denote a strategy for a follower j as $s_{f,j}=(s_{f,j}^t)_{t=1}^T \in S_{f,j}$, where $s_{f,j}^t$ is a particular decision of follower j at time t, and $S_{f,j}$ is the set of alternative strategies. A combination of strategies adopted by N leaders for the entire duration of the game can be presented as $s_l=(s_{l,1},\dots,s_{l,N})\in S_l$, and for M followers as $s_f=(s_{f,1},\dots,s_{f,M})\in S_f$, where the decision spaces S_l, S_f are defined as

$$S_l=\prod_{i=1}^{N} S_{l,i} \quad \text{and} \quad S_f=\prod_{j=1}^{M} S_{f,j}.$$

By optimizing the objectives of the competing firms in these Stackelberg games, we obtain the optimal strategies for the leaders and followers as the solution.

Definition (*General Stackelberg competition model*). A general multi-leader–follower Stackelberg competition model with N leaders and M followers may be formulated as

$$\max_{s_l,s_f} \quad \sum_{i=1}^{N} \Psi_{l,i}(s_l,s_f) \tag{1}$$

$$\text{s.t.} \quad s_f=\arg\max_{s_f}\left\{\sum_{j=1}^{M} \Psi_{f,j}(s_l,s_f) : g_{f,h}(s_l,s_f)\le 0, h\in\{1,\dots,H\}\right\}, \tag{2}$$

$$g_{l,k}(s_l,s_f)\le 0, \quad k\in\{1,\dots,K\}, \tag{3}$$

$$s_l\in S_l, \quad s_f\in S_f, \tag{4}$$

where $\Psi_{l,i},\Psi_{f,j}$ denote the objective functions for leader i and follower j respectively. Similarly, the constraints for the leaders and followers are given by mappings $g_{l,k}$ and $g_{f,h}$. The number of constraints for the leaders and followers is K and H respectively.

Each of the N leaders acts as a traditional Stackelberg leader towards the follower firms, but as a Cournot firm with respect to the other $N-1$ leaders. The N leaders choose their strategies simultaneously and non-cooperatively among themselves, as in a classic Cournot competition model, but collectively they are in a Stackelberg competition with the follower firms. Similarly, the M followers act as Stackelberg followers towards the leaders and as Cournot firms with respect to each other. Thus, the model consists of two Cournot competitions encompassed by a Stackelberg competition model. The generality of the formulation allows us to easily extend the problem to include as many leaders and followers as necessary, and to make them as different from one another as needed to model a real-world market situation. However, in this paper we assume that the N leaders and M followers are symmetric among themselves.

In the next section, we present a more concrete example of this general model. Additional complexity is introduced into this extended model by incorporating investment and marketing variables into each decision, expenditure constraints, and discrete production variables into the model for both the leaders and the followers.