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A robust evolutionary algorithm for the recovery of rational Gielis curves

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ABSTRACT

Gielis curves (GC) can represent a wide range of shapes and patterns ranging from star shapes to symmetric and asymmetric polygons, and even self intersecting curves. Such patterns appear in natural objects or phenomena, such as flowers, crystals, pollen structures, animals, or even wave propagation. Gielis curves and surfaces are an extension of Lamé curves and surfaces (superquadrics) which have benefited in the last two decades of extensive researches to retrieve their parameters from various data types, such as range images, 2D and 3D point clouds, etc. Unfortunately, the most efficient techniques for superquadrics recovery, based on deterministic methods, cannot directly be adapted to Gielis curves. Indeed, the different nature of their parameters forbids the use of a unified gradient descent approach, which requires initial pre-processings, such as the symmetry detection, and a reliable pose and scale estimation. Furthermore, even the most recent algorithms in the literature remain extremely sensitive to initialization and often fall into local minima in the presence of large missing data. We present a simple evolutionary algorithm which overcomes most of these issues and unifies all of the required operations into a single though efficient approach. The key ideas in this paper are the replacement of the potential fields used for the cost function (closed form) by the shortest Euclidean distance (SED, iterative approach), the construction of cost functions which minimize the shortest distance as well as the curve length using R-functions, and slight modifications of the evolutionary operators. We show that the proposed cost function based on SED and R-function offers the best compromise in terms of accuracy, robustness to noise, and missing data.

1. Introduction

When one tries to model nature, one faces a trade-off between the number of degrees of freedom (DoF) of a chosen model and the difficulty to determine its parameters: the higher the number of DoF, or dimensionality of the research space, the more powerful and versatile is the representation, but also the more difficult it is to retrieve its parameters to fit a given data set. In the literature, numerous techniques exist to represent various types of curves and surfaces, with their own strengths and weaknesses depending on the application and the nature of the data to be modeled. To represent simple curves or surfaces, one strategy consists in using deformable closed objects, such as circles or spheres for example. The researches dedicated to superellipses, superquadrics, and their extensions belong to this category. Superquadrics have been extensively studied since their introduction in 1981 in the field of computer graphics [1]. Superquadrics have found numerous applications due to their limited number of shape parameters and their ability to represent a wide diversity of simple objects ranging from diamonds, cubes, cylinders, spheres, and intermediate shapes. Unfortunately, superquadrics are not versatile enough to represent very common natural shapes, especially due to their fixed symmetries and their incapacity to represent asymmetrical shapes. These issues have been overcome by Gielis et al. [2,3], who introduced a parametric formulation for generalized circles or ellipses and which can represent regular polygons as well as self intersecting and asymmetric curves. Unfortunately, the methods dedicated to superquadrics recovery are not directly adaptable to Gielis curves and require several pre-processings, mostly symmetry detection and acceptable initialization. The techniques for the recovery of deformable models have been widely applied in computer vision [4] and are very often based on gradient descent approaches. Among them, the Levenberg–Marquardt algorithm is one of the most widely used and is considered as the standard method for superquadrics recovery [5,6]. However, this algorithm requires the optimized parameters to be differentiable, which is not the case for Gielis curves, for which the symmetries are natural integers or rational numbers. Additionally, the parameter estimates for these methods become unreliable for partial data, which requires several improvements as proposed for 2D curves in [7]. Another fundamental difficulty for these methods is the definition of an appropriate cost function to be minimized. Traditionally, the cost function is built

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upon a signed potential field. Various cost functions and recovery strategies for superquadrics have been proposed in the literature. For instance, Jaklic et al. directly use the superquadrics potential field [5], whereas Gross and Boult introduced the radial distance in [8]. An experimental comparison of these approaches is found in [9]. Inspired by these previous works, we have proposed several potential fields, combination strategies, and associated cost functions for Gielis curves and surfaces in [10–13].

Since the number of symmetries for Gielis curves is not differentiable, a symmetry detection step is required prior to the reconstruction. When the symmetries are not correctly detected, *e.g.* for incomplete data, these algorithms often fall into local minima. In order to overcome this fundamental issue, a stochastic approach using evolutionary algorithms (EA) has been proposed in [10], and has been shown to be able to simultaneously recover multiple Gielis surfaces from unsegmented cloud of points. Nevertheless, several aspects of this approach have not been studied, and the execution time remains far beyond acceptable durations due to the high number of dimensions of the research space. For instance, if EAs are known for their ability to converge to the global optimum, the development and the setting of their inner parameters, such as the number of crossings, the population size, the mutation rate, etc., are very often left to the user skills. The algorithm presented in this paper is an improvement of the one proposed by Bokhabrine et al. [10], though restricted to its simplest formulation, namely the reconstruction of 2D Gielis curves. Hence, in this paper, we reduce the complexity of the problem for a simpler framework in which the accuracy, the efficiency, and the robustness of the proposed algorithm can be clearly analyzed and justified. We present slight modifications of the traditional EAs and analyze the robustness of the proposed algorithm to noise and data incompleteness. More specifically, we show that with our approach, the results are robust to strong variations of the critical parameters, such as the number of crossing points, the population size, and mutation rate. Moreover, by replacing the potential fields (closed form solution) proposed in [11,13] by the shortest Euclidean distance (iterative method), we allow for a stronger and better discrimination between the solutions close to a local optimum, which leads to better reconstruction results in which details such as small peaks are more correctly recovered. We also discuss several strategies to introduce the length of the recovered surface into the cost function and we show that the proposed technique, based on R-functions, avoids the degenerate cases where the curve collapses to a point or inflate to infinity, and provides the best compromise in terms of accuracy and robustness compared to other standard potential fields.

The structure of the rest of paper is as follows: in Section 2 we recall the initial parametric definition of Gielis curves, and their associated potential fields. In Section 3 we present our modified evolutionary algorithm to recover Gielis curves. We discuss several strategies for the definition of an appropriate cost function in Section 4. Section 5 describes our results under different noise regimes. Comparisons between the suggested potential fields with standard Levenberg–Marquardt algorithm are also provided. Conclusions and future work are presented in Section 6.

2. Gielis curves

Gielis et al. introduced the superformula which can be seen as the parametric radius of a generalized circle in [2,3]. In polar coordinates, the radius $r(\theta)$ of a Gielis curve is defined by

$$r(\theta)=\frac{1}{\sqrt[n_1]{\left|\frac{1}{a}\cos\left(\frac{m\theta}{4}\right)\right|^{n_2}+\left|\frac{1}{b}\sin\left(\frac{m\theta}{4}\right)\right|^{n_3}}},\tag{1}$$

with a, b, and $n_i\in\mathbb{R}_*$, and $m\in\mathbb{R}_*^+$.

m represents the number of symmetry axis and can also be seen as the number of sectors in which the plane is folded. Parameters a and b control the relative scale over each sector. Coefficients n_1,n_2, and n_3 control the shape. Regular polygons can be generated by setting the shape coefficients to specific values as shown in [3]. For $m=4$ and $n_1=n_2=n_3$, the original superellipses are obtained.

Reconstruction algorithms are based on the mean-squared minimization of a suitable cost function built upon various GCs potential fields [12,10]. The construction of potential fields for GCs relies on the combination of pseudo radial distances obtained from any point P and the intersection I between the curve and the half-line [OP). When the curve is self intersecting, *i.e.* for rational Gielis curves (RGCs), this intersection is no longer unique and several potential fields should be combined. We have recently proposed a technique to build implicit fields for RGCs in [13]. In this case, by definition, the symmetry parameter m in Eq. (1) can be written as the ratio of two relative prime integers as $m=p/q$, and Eq. (1) becomes

$$r(\theta)=\frac{1}{\sqrt[n_1]{\left|\cos\left(\frac{p\theta}{4q}\right)\right|^{n_2}+\left|\sin\left(\frac{p\theta}{4q}\right)\right|^{n_3}}},\tag{2}$$

with $p,q\in\mathbb{N}_*^+$ relative prime numbers. The parameter p is similar to $m/1$, *i.e.* it still represents the symmetry number. The parameter q corresponds to the maximum number of self intersections, and the angle θ now belongs to $[0,2q\pi]$.

A 2D Gielis curve is defined in polar coordinates as

$$\begin{pmatrix}x(\theta)\\ y(\theta)\end{pmatrix}=\begin{pmatrix}(\theta)\cos\theta\\ r(\theta)\sin\theta\end{pmatrix}.\tag{3}$$

For a point $P(x,y)$, one can determine one intersection I between the curve and the half line $[OP)$ as $I=(r(\theta)\cos\theta, r(\theta)\sin\theta)$, with $\theta=\tan^{-1}(y/x)$, and we have $I^2=r^2$. From this definition, several potential fields can be defined such as the curve corresponds to their zero sets, the inside being (by convention) the set of points for which the field is positive, and the outside to the set of points for which the field is negative.

In [11], Fougerolle et al. define a potential field $F_1(x,y)$ from Eq. (3) as

$$F_1(x,y)=1-\frac{x^2+y^2}{r^2(\theta)}.\tag{4}$$

Following Gross et al. in [8], a signed potential field $F_2(x,y)$ can be built as the radial distance between a point $P(x,y)$ and the intersection point I as

$$F_2(x,y)=r(\theta)-\sqrt{x^2+y^2}.\tag{5}$$

As suggested by Voisin in [14], one can penalize the points contained within the curve. For instance, one can consider the potential field $F_3(x,y)$ defined as

$$F_3(x,y)=\log\left(\frac{x^2+y^2}{r^2\theta)}\right).\tag{6}$$

3. An evolutionary algorithm for the recovery of rational Gielis curves

Evolutionary algorithms (EAs) are stochastic search methods that use some key mechanisms inspired by biological evolution and have been successfully applied in many optimization problems in pattern recognition, such as classification in [15], clustering in [16], and graph matching in [17]. Each candidate of the population, or individual, represents a solution to the problem and the fitness (or cost) function determines its adaption to the environment.