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A comparative study of different approaches using an outranking relation in a multi-objective evolutionary algorithm

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A R T I C L E I N F O

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A B S T R A C T

This paper presents a comparative analysis of three versions of an evolutionary algorithm in which the decision maker's preferences are incorporated using an outranking relation and preference parameters associated with the ELECTRE TRI method. The aim is using the preference information supplied by the decision maker to guide the search process to the regions where solutions more in accordance with his/her preferences are located, thus narrowing the scope of the search and reducing the computational effort. An example dealing with a pertinent problem in electrical distribution network is used to compare the different versions of the algorithm and illustrate how meaningful information can be elicited from a decision maker and used in the operational framework of an evolutionary algorithm to provide decision support in real-world problems.

1. Introduction and motivation

The modeling of real-world problems generally requires the consideration of distinct axes of evaluation of the merits of potential solutions. Namely in engineering problems, aspects of operational, economical, environmental and quality of service nature are at stake. Therefore, mathematical models must explicitly address these multiple, incommensurate and often conflicting aspects of evaluation as objective functions to be optimized. Besides this "realistic" reason, multi-objective optimization models enlarge the spectrum of potential solutions to be analyzed thus enabling to grasp the trade-offs between the objective functions that are relevant for decision purposes in order to reach a satisfactory compromise solution that can be accepted as the final outcome of the process. The essential concept in multi-objective optimization is the one of non-dominated (efficient, Pareto optimal) solutions, that is feasible solutions for which no improvement in all objective functions is possible simultaneously; in order to improve an objective function it is necessary to accept worsening at least another objective function value. In real-world problems, a high number of non-dominated solutions is likely to exist.

The use of Evolutionary Algorithms (EAs) to deal with multi-objective optimization (MOO) models has gained an increasing relevance due to their ability to work with a population of individuals (solutions) that hopefully converges to the true non-dominated front [1,2]. EAs are particularly suitable for tackling hard combinatorial and/or non-linear models, as they are less susceptible to the shape or continuity of the non-dominated front than the classical (mathematical programming) optimization methods. The rationale is that EAs deal with a population of solutions and the aim is generally the characterization of a non-dominated front. In this setting EAs incorporating techniques to preserve the diversity of solutions (for a comprehensive depiction of that front thus unveiling the trade-offs in different regions of the search space) possess advantages compared with the use of "scalarizing" functions, in which a surrogate scalar function aggregating the multiple objectives is optimized, as in traditional mathematical programming approaches. However, it must be noticed that, in real-world problems, this is, in general, "just" a potential non-dominated front, classified as such because no other solutions dominating it could be found but no theoretical tools exist, which guarantee their true Pareto optimality.

Although it is the essential concept in MOO, the concept of non-dominated solution is a poor one, in the sense that it lacks discriminative power for decision recommendation purposes. Non-dominated solutions are not comparable between them, so no solution arises as the "final" one [3,4]. The rationalization of the comparison between non-dominated solutions requires taking into account the expression of the decision maker's preferences that somehow "enrich" the non-dominance relation [5]. These preferences represent a set of opinions, values, convictions

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and perspectives of reality, which configure a personal model of the reality under study, which the decision maker (DM) leans on to evaluate the different potential solutions [6–8].

Recent studies have shown that EAs based on the non-dominance relation only are insufficient to deal with MOO models, namely whenever the number of objective functions is large [8–10]. In these situations, the non-dominance relation may become inefficient in the selection of individuals for the next generation and lead to a weak selective pressure [11,12]. As it is referred to in [13], in these cases the progress of the population tends to slow down and the time consumed in the search process to find, at least, a good approximation to the non-dominated front may become prohibitive. In addition to the problems related with the selection procedure and the time consumed in the search process, a major difficulty arises at the end of the optimization process when it is necessary to choose a solution (or a small set of solutions for further screening) having in mind its practical implementation. In fact, in a real-world MOO model, the number of solutions in the non-dominated front is generally very large due to the conflicting nature of the objective functions, possibly its number, and the frequent combinatorial nature of the problem [3].

The preference information supplied by the DM is of paramount importance to guide the search to the regions where solutions more in accordance with his/her preferences are located, thus narrowing the scope of the search to the regions of interest and reducing the computational effort [7,14,15]. The convergence to these regions is improved by incorporating mechanisms for preference expression into the evolutionary process. Therefore, techniques aimed at meaningfully capturing and incorporating the DM's preferences into the evolutionary process should play a key role in real-world decision processes based on complex (namely combinatorial) MOO models. The use of weights to capture the relative importance of the multiple objective functions by aggregating them in a single surrogate function is one of the most popular techniques to include preferences into an EA, which may be operationalized in diverse manners. The underlying idea in these "scalarizing" approaches is that the optimal solution to the single objective function is a non-dominated solution to the multi-objective model. However, attention shall be paid to the links between weights perceived as importance coefficients and scaling coefficients. In [16] the degrees of importance are defined in a way similar to the linguistic ranking methods and are then converted into real numbers that can be used as objective function weights in an EA. Jim and Sendhoff [17] convert fuzzy preferences into interval-based weights and use them in a dynamic weighting aggregation method. Another well-known approach for the inclusion of preferences into EAs consists in using the concept of goal attainment or (minimization of a distance to a) reference point [18–20]. The knees of the Pareto-optimal front are considered by some authors as usual regions of interest and the incorporation of preferences is done to guide the search to those regions [15,21]. In other works the non-dominance relation is replaced or modified to include preferences into an EA [22–24].

The introduction of the preference expression parameters used in the ELECTRE TRI method has revealed to be suitable both from the point of view of meaningfulness of preference elicitation and its use in the operational framework of an EA [25]. The different versions of the EA developed (called EvABOR, Evolutionary Algorithm Based on an Outranking Relation) include features of the ELECTRE TRI method to guide the search according to the preference information expressed and use an outranking relation and the concept of classes of merit to generate the population for the next generation. Preferences are herein represented by means of technical parameters: weights, indifference, preference and veto thresholds, a set of references profiles and a cutting level (which may be perceived as the level of exigency of the classification). The weights reflect the true importance of each objective function (its "voting power") and are not scaling coefficients to achieve some aggregate value. The veto threshold enables to preclude situations often arising in real-world problems in which full compensation between the objective function values is undesirable or even unacceptable. The indifference and preference thresholds enable to introduce a gradual preference relation. The reference profiles define the classes of merit in which the solutions are classified, as explained in the next section, and the aim of the EvABOR approaches is to obtain solutions belonging to the best class of merit as much as possible.

Broadly, the incorporation of the DM's preferences may be done before (*a priori*), during (progressive) or after (*a posteriori*) the optimization process is carried out [5,6,26]. The incorporation of preferences into the EA using the ELECTRE TRI method may also be done in this manner. The *a posteriori* approach has been implemented and the obtained results have been compared with the first *a priori* version in [27]. The *a posteriori* approach produced significantly worse results motivating the development of the EvABOR algorithm in an *a priori* approach (having in mind a possible progress to an interactive approach). The three versions of EvABOR presented and compared in this paper have been developed to analyze the symbiosis between the outranking and the dominance relations in more detail with the aim to provide an answer to a set of relevant questions for ensuring the effectiveness of the search, such as: which must be the priority relation, the dominance or the outranking relation? How can the algorithm lead the evolutionary process to the region of interest (according to the preferences elicited) more efficiently? Is it advantageous to pass to the next generation all the solutions in the best class of merit or is it preferable to use non-dominated solutions only?

The introduction and motivation to this work have been presented in this section. In Section 2 the ELECTRE TRI method is briefly described. The EA endowed with features of the ELECTRE TRI method is presented in Section 3. The case study in electrical distribution networks is described in Section 4. Some illustrative results are analyzed in Section 5. Conclusions as well as some possible directions for future research are provided Section 6.

2. The ELECTRE TRI method

The ELECTRE TRI method belongs to the ELECTRE (*ELimination Et Choix Traduisant la REalité*) family of methods, which are based on the construction and exploitation of an outranking relation with respect to the problem to be tackled [28,29]. This is accomplished by performing comparisons between a pair of alternatives (solutions in the EA context) to establish preference, indifference or incomparability on the basis of all relevant information.

The ELECTRE methods may be classified accordingly to the type of the problem each one deals with: choice (to select the best alternative, or a set of the best alternatives), sorting (to assign each alternative to predefined ordered classes) and ranking (to establish a partial or complete pre-order of the alternatives). The ELECTRE TRI method is aimed at dealing with the sorting problem. In choice and ranking problems the alternatives are compared against each other, while in the sorting approach the comparisons are made between the alternatives and a set of alternatives defined by the DM (reference profiles). This aspect presents two important advantages. Since the number of reference profiles is in general much lower than the number of alternatives, significant fewer comparisons must be done. The second aspect is concerned with the quality of the alternatives. In the ranking problem the set of solutions is partially or completely ordered; however, the quality of all solutions may not be very