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Discrete Optimization

An efficient evolutionary algorithm for the ring star problem ☆

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ABSTRACT

This paper addresses the ring star problem (RSP). The goal is to locate a cycle through a subset of nodes of a network aiming to minimize the sum of the cost of installing facilities on the nodes on the cycle, the cost of connecting them and the cost of assigning the nodes not on the cycle to their closest node on the cycle. A fast and efficient evolutionary algorithm is developed which is based on a new formulation of the RSP as a bilevel programming problem with one leader and two independent followers. The leader decides which nodes to include in the ring, one follower decides about the connections of the cycle and the other follower decides about the assignment of the nodes not on the cycle. The bilevel approach leads to a new form of chromosome encoding in which genes are associated to values of the upper level variables. The quality of each chromosome is evaluated by its fitness, by means of the objective function of the RSP. Hence, in order to compute the value of the lower level variables, two optimization problems are solved for each chromosome. The computational results show the efficiency of the algorithm in terms of the quality of the solutions yielded and the computing time. A study to select the best configuration of the algorithm is presented. The algorithm is tested on a set of benchmark problems providing very accurate solutions within short computing times. Moreover, for one of the problems a new best solution is found.

1. Introduction

The Ring Star Problem (RSP) arises in telecommunications network design problems where a subset of nodes is selected for installing concentrators. The selected nodes are interconnected by a ring network (this is known as ring topology), whereas the remaining nodes are linked to their closest concentrator (this is known as star topology). The RSP can also model location–allocation problems occurring in logistics where some retailer locations are used as small depots served from a central depot in a single-vehicle route. The remaining retailers are directly served from their nearest small depot.

Let $G = (V, E \cup A)$ be a mixed graph, where $V = \{0, 1, \ldots, n\}$ is the node set, $E = \{[i,j]: i, j \in V\}$ is the edge set and $A = \{(i,j): i, j \in V\}$ is the arc set. Node 0 is a distinguished node which is referred to as the depot or the root. Edges in E refer to undirected links which are used to form the ring structure and arcs in A refer to directed links used in the star structure. We assume that there is a nonnegative facility cost p_i associated with each node i. This may represent, for instance, the cost of placing a facility at node i. There is also a nonnegative ring cost c_{ij} associated with each edge $[i,j]$, representing the cost of connecting nodes i and j, and a nonnegative assignment cost d_{ij} associated with each arc (i,j), referring to the cost of node i being connected to node j. The RSP consists of selecting a subset of nodes $V' \subseteq V$, including the depot, where facilities are installed and interconnected by a cycle structure. The remaining nodes are each connected to one of the facilities (see Fig. 1). The goal is to minimize the total cost, being the sum of the cost of placing the facilities at the nodes of the cycle plus the cost of the ring connections and the assignment costs. The cycle connection cost is the sum of all edge costs on the cycle. The assignment cost is defined as $\sum_{i \in V \setminus V'} \min_{j \in V'} d_{ij}$.

To formulate the RSP, we define

$$z_i = \begin{cases} 1, & \text{if } i \in V \text{ is on the cycle} \\ 0, & \text{otherwise} \end{cases}$$

$$x_{ij} = \begin{cases} 1, & \text{if edge } [i,j] \in E \text{ is on the cycle} \\ 0, & \text{otherwise} \end{cases}$$

$$y_{ij} = \begin{cases} 1, & \text{if node } i \in V \text{ is assigned to node } j \text{ on the cycle,} \\ & (i,j) \in A \\ 0, & \text{otherwise} \end{cases}$$

In order to simplify the notation, we denote $\{z_i, i \in V; x_{ij}, [i,j] \in E; y_{ij}, (i,j) \in A\}$ by $\{z, x, y\}$.

Then, the RSP can be formulated as the following binary problem:

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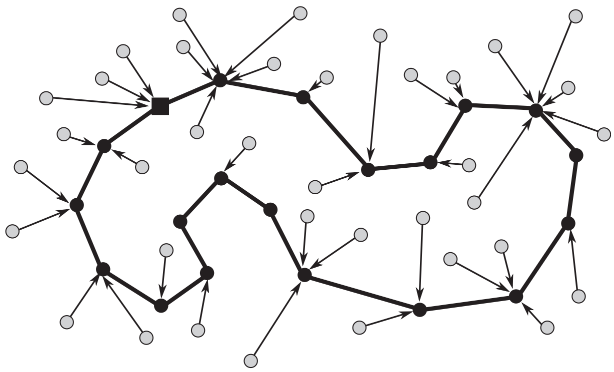

Fig. 1. A feasible solution of the RSP.

$$\min_{z,x,y} \sum_{i\in V} p_i z_i + \sum_{[i,j]\in E} c_{ij} x_{ij} + \sum_{(i,j)\in A} d_{ij} y_{ij} \quad (1a)$$

$$\text{subject to} \sum_{[i,j]\in E} x_{ij} = 2z_i, \quad \forall i \in V \quad (1b)$$

$$\sum_{(i,j)\in A} y_{ij} = 1 - z_i, \quad \forall i \in V \quad (1c)$$

$$\sum_{[i,j]\in E(S)} x_{ij} \leqslant |S| - 1, \quad \forall S \subseteq V \setminus \{0\}, \quad S \neq \emptyset \quad (1d)$$

$$z_0 = 1 \quad (1e)$$

$$z_i \in \{0,1\}, \quad \forall i \in V \quad (1f)$$

$$x_{ij} \in \{0,1\}, \quad \forall [i,j] \in E \quad (1g)$$

$$y_{ij} \in \{0,1\}, \quad \forall (i,j) \in A \quad (1h)$$

where $E(S) = \{[i,j] \in E: i, j \in S \subseteq V\}$. In this formulation, the objective function (1a) minimizes the total cycle and assignments costs, and the cost of placing the facilities at the nodes. Constraints (1b) enforce that the degree of each node i is 2 if and only if it belongs to the cycle. Constraints (1c) ensure that either each node i is on the cycle or is assigned to a single node j on the cycle. Constraints (1d) are the well-known subtour elimination constraints limiting the number of cycles to one. Constraint (1e) ensures that the depot is on the cycle. Constraints (1f)–(1h) guarantee that all variables are binary.

The RSP is also known as the first version of the Median Cycle Problem (MCP1). Under this name, Moreno Pérez et al. [22] use the model to test a heuristic that combines variable neighborhood search and tabu search using a set of benchmark problems based on Traveling Salesman Problem (TSP) instances from TSPLIB [24] involving between 50 and 200 nodes. In order to provide an optimal solution of the RSP, the algorithms developed in the literature mainly consider branch and cut techniques. Labbé et al. [17] formulate the problem as a mixed integer linear program, which uses connectivity constraints instead of constraints (1d), and relaxes some integer requirements of variables $\{z_i\}_{i\in V}$ and $\{y_{ij}\}_{(i,j)\in A}$ in formulation (1). After a polyhedral analysis of the problem, they propose a branch-and-cut algorithm and apply it to solve the benchmark problems and some randomly generated instances with a number of nodes between 10 and 300. Kedad-Sidhoum and Nguyen [16] propose a new formulation of the RSP based on chains and use it to develop a new branch-and-cut approach. A branch-and-cut algorithm is also proposed by Simonetti et al. [26], who reformulate the RSP as a minimum Steiner arborescence problem. They provide an optimal solution for some benchmark instances not solved in [17]. In summary, an optimal solution has been achieved for 120 out of the 124 benchmark instances considered in the literature [17,26].

Besides the above mentioned heuristic, other heuristic algorithms have been proposed in the literature. Renaud et al. [25] propose a multistart greedy add heuristic and a random keys evolutionary algorithm. Both algorithms are tested using only the benchmark instances which were exactly solved in [17] and proved to be quite efficient compared with [22]. Dias et al. [12] propose a heuristic that uses a general variable neighborhood search to improve the quality of the solution obtained by a greedy randomized adaptive search procedure. They only solve the benchmark instances having between 50 and 100 nodes, obtaining better results than [22] in some cases.

Other variants of the RSP have been introduced in the literature. Baldacci et al. [3] introduce the capacitated m-ring-star problem which consists of designing a set of m rings with bounded capacity that passes through the depot and through some transition points and/or customers, and then assigning each non-visited customer to a visited point or customer. A branch-and-cut approach is proposed to solve the problem. For the same problem, Naji-Azimi et al. [23] develop a heuristic algorithm which follows the scheme of the Variable Neighborhood Search and incorporates an Integer Linear Programming based improvement. Baldacci and Dell'Amico [2] generalize the above problem by allowing the existence of multiple depots. Finally, Liefooghe et al. [19] propose to consider individually on the one hand the costs of the ring and on the other hand the assignment costs. Based on this bi-objective formulation they propose different metaheuristics for solving the problem.

The goal of this paper is to propose an evolutionary algorithm for the RSP based on a new encoding scheme to handle which nodes are on the ring. This algorithm has been suggested by a new formulation of the RSP as a bilevel problem with three decision makers, a leader and two independent followers. The paper is organized as follows. In Section 2 the formulation of the RSP as a bilevel problem is proposed and the main results of the equivalence between both problems are proved. The algorithm is developed in Section 3. In Section 4 the computational performance of the procedure is evaluated using the benchmark instances dealt with in the literature. Finally, Section 5 concludes the paper with some final remarks.

2. A bilevel formulation for the RSP

Bilevel programming has been proposed for modeling hierarchical processes characterized by the existence of two decision levels. The decision makers at both levels of the hierarchy seek to optimize their individual objective functions and control their own set of decision variables. Due to the hierarchical structure of the process, the decision maker at the upper level of the hierarchy, also called the leader, aims to optimize his own objective function but anticipating within the optimization scheme the reaction of the decision maker at the lower level, also called the follower. In mathematical terms, the bilevel programming problem involves two optimization problems where the constraint region of the upper level optimization problem is implicitly determined by the lower level optimization problem. Bilevel programming is discussed in Bard [4], Colson et al. [10] and Dempe [11]. Some extensions of bilevel programming consider the existence of several decision makers at the lower decision level [6] or multiple objectives at each decision level [7,8].

In the RSP, the decision maker has to locate a simple cycle and assign the remaining nodes to the nodes on the cycle in an optimal way. In the bilevel programming formulation that we propose, the decision maker will share the decision process. This decision maker will act as a leader and delegate some of the decisions to two followers. The idea is that the leader will decide on the nodes of the cycle, but anticipating the reactions of both followers. Each follower, after receiving the leader's selection, solves his own problem. One follower will find the cycle by solving a traveling