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# Evolutionary algorithm assisted by surrogate model in the framework of ordinal optimization and optimal computing budget allocation ☆

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ABSTRACT

This work proposes an evolutionary algorithm (EA) that is assisted by a surrogate model in the framework of ordinal optimization (OO) and optimal computing budget allocation (OCBA) for use in solving the real-time combinatorial stochastic simulation optimization problem with a huge discrete solution space. For real-time applications, an off-line trained artificial neural network (ANN) is utilized as the surrogate model. EA, assisted by the trained ANN, is applied to the problem of interest to obtain a subset of good enough solutions, S. Also for real-time application, the OCBA technique is used to find the best solution in S, and this is the obtained good enough solution. Most importantly, a systematic procedure is provided for evaluating the performance of the proposed algorithm by estimating the distance of the obtained good enough solution from the optimal solution. The proposed algorithm is applied to a hotel booking limit (HBL) problem, which is a combinatorial stochastic simulation optimization problem. Extensive simulations are performed to demonstrate the computational efficiency of the proposed algorithm and the systematic performance evaluation procedure is applied to the HBL problem to quantify the goodness of the obtained good enough solution.



## 1. Introduction

In recent years, research into methods for solving real-world *expensive optimization problems* in the field engineering has been increasing. These include air foil shape optimization, computer-aided design (CAD) problems, the computational electromagnetic problem, the stochastic simulation optimization problem and the combinatorial optimization problem with a computationally expensive objective function. Evolutionary algorithms (EAs) that are assisted by surrogate models [2,19,20,25–27,35] provide possible solutions to such problems, and various versions have been proposed [7,14,33,36,38,40,43]. Nain and Deb used coarse-to-fine surrogate models [33] to improve the computational efficiency and the quality of the solution. Schmit and Lipson utilized the coevolution technique to handle simultaneously the approximation level of surrogate model and the accuracy of the fitness predictor [36]. Zhou *et al.* presented a memetic algorithm that combines global and local surrogate models to accelerate evolutionary optimization [43]. Tenne and Armfield developed a framework of memetic optimization [38] using variable global and local surrogate models based on the radial basis

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function (RBF) networks to solve the airfoil shape optimization problem. Guimaraes et al. proposed a memetic algorithm that combined two strategies, which were varying the accuracy in black-box functions and approximation-based local search, to solve a CAD problem that involved an electromagnetic device [14]. Ting and Liao used a memetic algorithm to solve a set k-cover problem to extend the wireless-sensor network lifetime [40]. Chen et al. developed a hybrid algorithm to solve resource-constrained project scheduling problems in real time [7].

In the above existing methods [14,20,26,33,35,36,38,43], surrogate models are typically constructed on-line, based on exact evaluations of objective function, to ensure accuracy. However, to reduce computing time, only a few exact evaluations of objective functions are performed, potentially yielding a coarse global model and introducing false optima owing to the failure to capture the problem landscape, as noted by Jin [18]. More exactly evaluating objective functions to eliminate this problem will increase computing time. Accordingly, increased computing time is the price paid for the increased accuracy of the on-line built surrogate model. Indeed, reducing computational time is the main purpose of most of the above mentioned methods [7,14,20,26,33,35,36,38,40,43], but most of them, with the exception of that developed by Chen el al. in [7] have been applied to engineering design problems, which do not require application in real-time.

This work concerns a *real-time combinatorial stochastic simulation optimization problem* with a *huge discrete solution space*. The computing budget for solving real-time application problems is generally very limited. Numerous sample points are usually required to construct the surrogate model for a huge discrete solution space. These two characteristics prevent the on-line construction herein of the surrogate model for the problem of interest. A model that is constructed off-line may not be inaccurate but its accuracy requires justification, such as an error analysis, as will be demonstrated in this paper. Because of the limited computing budget, optimality in the real-time application problem is normally traded for a "good enough" solution that can be obtained in real-time. The objective of the ordinal optimization (OO) that was proposed by Ho [16] is to take a limited computing time to find a good enough solution to an expensive optimization problem. The basic idea behind OO is as follows. Let $S$ denote the set of *estimated* good enough designs that are selected from $N$ given designs using a surrogate model, and let $G$ denote the subset of *actual* good-enough designs, such as the top $n\%$ of the $N$ designs; now, the probability that $S$ contains at least ($k \geqslant 1$) elements of $G$ is high. Hence, the *first phase* of OO is the use of a computationally efficient *surrogate model* to evaluate the $N$ given designs and to select the estimated top $|S|$ designs, where $|(\bullet)|$ denotes the cardinality of ($\bullet$). The *second phase* is to use the time-consuming *exact model* to evaluate each of the selected designs in $S$, of which the top $k$ designs will be the actual good enough solutions with high probability, $p_k$. OO theory provides a quantitative result for the performance, represented by a relationship among $|S|$, $p_k$, $k$, $n$, $N$ and $\zeta$ [23,28], where $\zeta$ represents the magnitude of the error in the surrogate model. This theoretical result determines the value of $|S|$ for the given $p_k$, $k$, $n$, $N$ and $\zeta$. In general, the goodness of the obtained good enough solution is proportional to $|S|$. However, a larger $|S|$ can be obtained only with a larger computing time in the second phase of OO. Therefore, the desired goodness of the solution can be determined from the computing budget. This theoretical result distinguishes OO from ordinary heuristic algorithms.

However, when the discrete solution space is huge rather than a limited $N$, evaluating each solution, even using the surrogate model in the first phase of OO, is unacceptable. To overcome this difficulty, EA can be used with a computationally efficient surrogate model to search the estimated good enough subset of solutions, $S$. However, the existing theoretical result obtained using OO cannot be applied in the new approach. Furthermore, to accelerate the computation in real-time applications, an off-line trained artificial neural network (ANN) will be utilized herein as the surrogate model in the first phase of OO and the optimal computing budget allocation (OCBA) technique [5,6] will be adopted to identify the best solution in $S$ in the second phase of OO. The use of this technique replaces the use of the exact model to evaluate each solution in $S$. Notably, the applications of the proposed algorithm require only one good enough solution.

Analyzing the performance of EA that is assisted by the off-line trained ANN is difficult, as is the follow up step of using the OCBA technique to find the best solution in $S$. Even the performance of sophisticated EAs that are assisted by surrogate models [33,36] and the successful memetic algorithms [7,14,38,40,43] are investigated by comparison with other existing methods by simulations of benchmark test problems. Therefore, this work makes three main contributions. The *first* is the development of an EA that is assisted by a surrogate model in the framework of OO and OCBA for finding a *good enough solution* of the real-time combinatorial stochastic simulation optimization problem using *limited* computing time. The *second* is the application of the proposed approach to solve the hotel booking limits (HBLs) problem, which is a popular combinatorial stochastic simulation optimization problem with a huge solution space [37]. The *third* is the development of a systematic procedure for analyzing the performance of the proposed approach.

This paper is organized as follows: Section 2 describes the problem of interest. Section 3 presents the proposed algorithm and the procedure for systematically analyzing its performance. Section 4 presents an application example – HBL problem, the application of the proposed algorithm to the HBL problem with test results, and an analysis of the computing budget required by OCBA. Section 5 presents a performance analysis of the proposed algorithm when applied to this example. Section 6 draws conclusion.

## 2. Problem statement

The real-time combinatorial stochastic simulation optimization problem of interest is stated as follows.

$$\min_{\theta \in \Theta} E[J(\theta)] \tag{1}$$