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Interactive evolutionary algorithms with decision-maker's preferences for solving interval multi-objective optimization problems

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ABSTRACT

Interval multi-objective optimization problems (IMOPs), whose parameters are intervals, are considerably ubiquitous in real-world applications. Previous evolutionary algorithms (EAs) aim at finding the well-converged and evenly-distributed Pareto front. An EA incorporating with a decision-maker (DM)'s preferences was presented in this study to obtain a Pareto-optimal subset that meets the DM's preferences. In this algorithm, the DM's preferences in terms of the relative importance of objectives were interactively input, and the corresponding preferred regions were then obtained. Based on these regions, solutions with the same rank were further distinguished to guide the search towards the DM's preferred region. The proposed method was empirically evaluated on four IMOPs and compared with other state-of-the-art methods. The experimental results demonstrated the simplicity and the effectiveness of the proposed method.

1. Introduction

Multi-objective optimization problems (MOPs) are ubiquitous in real-world applications, e.g., product design [1], path planning [2] and power dispatch [3]. These optimization problems contain more than one objective and there exist some conflicts among these objectives, indicating that there is no solution which is optimal for all objectives. In addition, uncertain parameters, e.g., stochastic variables, fuzzy numbers, and intervals, are often presented in the above problems, which are called uncertain MOPs.

When the uncertain parameter is a stochastic variable, the probability distribution of the stochastic variable needs to be known in advance; when it is a fuzzy number, the membership function that specifies the fuzzy number should be given *a prior*. However, it is difficult to obtain either the accurate probability distribution or appropriate membership function in practice. When the uncertain parameter is an interval, the upper and lower bounds or the midpoint and the radius of the interval need to be known beforehand. IMOPs are focused on in this paper since the value of the interval parameter is easy to be acquired.

There have been few theories and methods suitable for IMOPs so far, since they are complicated in practical applications. The reasons include the following: (1) IMOPs involve multiple objectives; (2) these objectives are uncertain. Both of these two points indicate that the effective solutions of IMOPs are meaningful.

The goal of an IMOP is to find one (or several) DM's most preferred solution(s). Therefore, it is necessary to incorporate with her/his preferences. Generally speaking, there are three approaches to embed the DM's preferences [4]: The first one is *a priori* methods. The DM needs to pre-specify her/his preference information, e.g., a set of weights on different objectives. However, it is difficult for the DM to provide accurate preferences when her/his cognition on an optimization problem is insufficient, resulting in a suboptimal solution. The second one is *a posteriori* methods. A Pareto-optimal set is first generated by a multi-objective optimization method, and then the DM chooses one (or several) solution(s) from this set. Since a large number of solutions may be generated, and among which many are not preferred by the DM. With many candidates to select, her/his burden in selection is increased. The third one is interactive methods. The DM's preferences are incorporated into the course of optimization to select satisfactory solution(s). They have the following three advantages [5]: (1) preference information requested from the DM is usually much simpler than that required by *a priori* methods; (2) they have moderate computational requirements in comparison to *a posteriori* methods; (3) as the DM gets more involved in the search, and will be more confident about the final choice. Since interactive methods are very promising in solving practical MOPs, they are used to solve the IMOP in this study.

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In this study, we proposed a preference-based EA for IMOPs by employing NSGA-II, and the DM's preferences were incorporated by interactive methods. A preliminary version of this algorithm is proposed in [6]. In this paper, we extend the previous work in the following directions:

(1) Providing the related work.
(2) Presenting detailed algorithm description, such as, the specific mathematical models of the preferred regions, i.e. inequalities (3)–(7) were provided; two examples were used to explain how to judge a solution's location in the third paragraph of Section 3.2; formula (8) was extended to the case when the interval objective degenerates to a point interval, i.e., formula (10).
(3) Giving the steps of the proposed method and its complexity analysis.
(4) Employing a new performance indicator, i.e., imprecision, to measure the uncertainty of the Pareto-optimal set obtained by the proposed method.
(5) Extending the experiments to investigate the influence of different settings of some parameters on the proposed method. These parameters contain α which is used to distinguish the type of the DM's preferences, the degree and the frequency of these preferences' change, and the chance of using the anaphase preferences. In addition, we further analyzed the experimental results.
(6) Conducting a non-parametric statistical test, i.e., Mann–Whitney U test, to analyze the experimental results.

The primary contributions of this study can be summarized as follows. First, the notion of relative importance of objectives in reference [7] was further extended. Second, a method was presented to map the relative importance of objectives to the corresponding preferred regions of the objective space. Third, a criterion incorporating the DM's preference to choose solutions with the same rank was presented to steer the search to the DM's preferred region.

The remainder of this paper is organized as follows. Section 2 reviews related work. Section 3 presents the proposed method. The application of the proposed method in typical IMOPs and comparative experiments are given in Section 4. Finally, Section 5 concludes this paper and offers suggestions on possible opportunities for future research.

2. Related work

Without loss of generality, the following IMOP is considered:

$$\max \quad f(\mathrm{x},\mathrm{c}) = (f_1(\mathrm{x},\mathrm{c}), f_2(\mathrm{x},\mathrm{c}), \ldots, f_m(\mathrm{x},\mathrm{c}))$$
$$\text{s.t} \quad \mathrm{x} \in S \subset R^n$$

$$\mathrm{c} = (c_1, c_2, \ldots, c_l)^T, \quad c_k = [\underline{c}_k, \overline{c}_k], \quad k = 1, 2, \ldots, l \tag{1}$$

where x represents an n-dimensional decision variable; $f_i(\mathrm{x},\mathrm{c})$, $i = 1, 2, \ldots, m$ is the i-th objective with interval parameters; c is an interval vector parameter; and c_k is the k-th component of c with $\underline{c}_k$ and $\overline{c}_k$ being its lower and upper limits, respectively. Since the objectives contain interval parameters, their values in formula (1) are all intervals. In the context of different scales of different objectives, these scales can be normalized to make them identical [8].

During the last decades, intelligent algorithms are widely applied to many optimization problems. For example, particle swarm optimization algorithms are a kind of popular intelligent algorithms, and are used to optimize multimodal functions [9] and the structure of radial basis probabilistic neural networks [10]. While EAs are a kind of stochastic global optimization methods inspired by nature evolution and genetic mechanisms. In the following, the related works on EA are presented.

Few works on IMOPs have been obtained up to date. Aiming at a MOP whose objectives contain noises, a multi-objective EA (IP-MOEA, for short) was proposed by Limbourg and Aponte. In this method, the objective values are expressed with intervals, and two criteria, i.e., dominant relationship and hypervolume based on intervals, are used to measure the quality of a solution and the performance of a Pareto-optimal set, respectively [11]. In our recent work, dominant relationship between two solutions and crowding distance of a solution suitable for interval objectives are defined and utilized to select optimal solutions, leading to solutions with small uncertainty and uniform distribution [12].

The above methods aim at finding a Pareto-optimal set with good convergence and uniform distribution. In practice, finding an approximate Pareto-optimal set is only one task of multi-objective optimization. To get a small number of satisfactory solutions of the DM, another task, i.e., decision-making, is also considerably important. As mentioned above, interactive methods have advantages of both *a priori* and *a posteriori* ones, so we will incorporate the DM's preferences into the evolution.

There are many interactive multi-objective EAs for deterministic MOPs and they differ basically from each other in what kind of information is requested from and shown to the DM at each iteration [13]. The types of the DM's preference information include reference points [13], reference directions [14], relative importance of objectives [7], and classification of objectives [15], where relative importance of objectives is formalized as strict preference, equality of importance, and incomparability, indicating that it can be described with languages. It is thus convenient for the DM to provide this type of preference information, and it is used to solve IMOPs in this study. It is worth noting that the methods proposed in this study and reference [7] differ in two ways. (1) Rachmawati et al. used this presentation and divided the prototype Pareto front into several preferred regions. Consequently, the mathematical model of relative importance of objectives is constructed [7]. In this study, preferred regions are obtained by dividing the objective space, and a mathematical model of relative importance of objectives is then deduced according to them in view of the idea of [7]. (2) A novel preference relation is included to the previous notion of relation importance of objectives in order to clearly express the DM's preferences.

Additionally, the interactive multi-objective EAs also differ from each other in what part of algorithms modified by the DM's preferences. These parts include objectives, dominance, and crowding distance [16].

For the objectives, Wagner and Trautmann specified the DM's preferences by using desirability function, and modified objective values in order to restrict the DM's preferred region [17].

For the dominance, in the method given by Deb et al. [18], a strictly increasing function is constructed to reflect a DM's preferences after the population has evolved for a number of generations set in advance. Then the function is used to compare solutions to guide the population to evolve towards the most preferred region. In the work of Shen et al. [19], a relationship of "assignment level superior" is constructed to replace the regular Pareto dominance, so as to distinguish the quality of solutions by quantifying the relative importance of objectives. A graphical user interface is used to allow the DM to modify her/his preferences in real time to adjust the search region. Branke et al. [5] proposed a guided MOEA, where the definition of dominance is modified based on the DM's preference. Said et al. [20] introduced a new variant of the Pareto dominance relation, called r-dominance, which has the ability to create a strict partial order among Pareto-equivalent solutions. This makes the above dominance able to guide the search toward the DM's most preferred region. Molina