A new mathematical formulation and fast algorithm for fully resolved simulation of self-propulsion

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ABSTRACT

We present a computational algorithm for fully resolved numerical simulation (FRS) of rigid and deforming bodies moving in fluids. Given the deformation of the body in its own reference frame, the method solves for the swimming velocity of the body together with the surrounding flow field, and the hydrodynamic forces on the body. We provide the mathematical foundation of the algorithm based on distributed Lagrange multipliers, and show that it naturally connects with vortex methods through a vorticity source at the interface. We demonstrate applications to rigid and flexible bodies, membranes, and bodies with a propelling membrane attached to them. In contrast to some existing methods, the swimming velocity of the body is not prescribed but is computed along with the forces, without requiring a body-fitted grid. The algorithm is designed to be fast, efficient, and easy to implement in existing fluid dynamics codes for practical solid–fluid problems in engineering and biology.

1. Introduction

Swimming fish are a biological model system for understanding the interconnections between mechanics, physiology and neuronal activity during locomotion. Musculoskeletal, neuronal, and sensory systems interact closely with each other resulting in the complex process of locomotion [1–3]. To gain insights into biolocomotion, a desirable approach is to use reduced order models for such multiple systems in an integrative setting to allow the study of how each function performs coherently along with the others. The empirical data or numerical simulations required for development and validation of reduced order models can become complex and expensive due to the detailed mechanics and neural function of the organism. In particular, the hydrodynamics of aquatic locomotion involves multiple phenomena such as viscous effects, boundary layer separation and vortex shedding which require elaborate measurements or computations to characterize sufficiently. Perhaps due to this complexity, many unanswered questions still remain, such as (i) how can the swimming velocity of an organism be determined from the movement of its propulsive surfaces? (ii) What is the efficiency of various modes of swimming? (iii) What are the wake signatures of the various modes of swimming, and are they indicative of the swimming efficiency?

A standard reduced order model approach relies on equating *a priori* estimates of drag and thrust on a fish at its swimming velocity. This has led to many conflicting results. For instance, for a given swimming velocity, the drag on swimming fish is often estimated to be higher than that found in towing experiments [4,5]. However, there are also measurements on an actively swimming robotic vehicle which show that the power needed to self-propel the robot is reduced by half compared to the power needed to tow the robot with the body straight and rigid [6]. Regarding the thrust at a given swimming speed,

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Lighthill's theory [5,7,8] is found to over predict thrust by a factor of up to 3. One of the root causes of the conflicting results on the drag and thrust on swimming fish is that in the case of self-propulsion, drag and thrust cannot be properly separated [4,9]. There is no net time-averaged hydrodynamic force acting on the surface of the fish during steady swimming. One may artificially split the net hydrodynamic force into two parts – thrust and drag. However, locations where thrust and drag are generated on the fish body can be intermingled [10]. Thus, there is no general way to *a priori* separate and predict both thrust and drag. This implies that the approach of balancing thrust and drag to obtain the swimming velocity is unsuitable. Fully resolved simulation (FRS) of self-propulsion is useful in this case to obtain the swimming velocity and at the same time obtain the force distribution on the surface of the fish. In our discussion, fully resolved simulation implies that the fluid–solid coupling is not modeled (e.g. using drag models) but instead the flow around the swimming body is fully resolved.

The separation of thrust and drag also produces difficulties in coming up with a consistent definition of swimming efficiency. Swimming efficiency is often given in terms of the Froude efficiency, which is defined as the ratio of work done by the thrust force to the total power input. Once again defining thrust is not straightforward when thrust and drag are intermingled as discussed above. As a result, there is no clear understanding of the comparative efficiency of the different swimming modes [4,10]. FRS of self-propulsion can provide the net mechanical energy cost of swimming and lead to useful efficiency measures such as those used by Kern and Koumoutsakos [11] to compare swimming efficiency across various swimming styles.

Usually, strong downstream flow in the wake is interpreted to signify thrust as well as energy lost in the wake [4,12–15]. However, the wake of steadily swimming eels is found to lack substantial downstream flow [10]. This raises a fundamental question: what is the signature of thrust in wakes of various swimming modes, and how is it correlated with swimming efficiency? Tytell and Lauder [10] speculate that wakes probably show a gradation from those of mackerel, for example, which primarily flap their caudal fins, to those of eels, which deform their entire body. The wake could thus be indicative of the propulsion mechanism which can be studied by FRS of self-propulsion. Such simulations will also make it possible to find swimming gaits that minimize wakes but are also efficient.

Each of the above issues could be addressed by sophisticated experimental techniques or through FRS schemes. Often experimental techniques are intrusive, making measurements in the natural state of the system difficult, especially when force measurements are involved [16]. Hence high-fidelity numerical simulations can greatly contribute to the study of the fluid dynamics of biolocomotion, leading to better predictive models for the motion of a variety of organisms.

Modeling the hydrodynamics of moving organisms can be challenging due to the strong coupling between the body and the surrounding fluid, which are governed by widely separate material characteristics such as stiffness and viscosity. To model this, there are two approaches of interest. In the first approach, referred to as the elastohydrodynamic approach, the muscle activation would be given. Then, the elastic equations for the body would be solved together with the equations of motion of the body and the fluid. This approach will require detailed knowledge of muscle anatomy, physiology, and muscle activation patterns, as well as the elastic properties of the fish body and fins. This requirement is far beyond what is understood for the best characterized systems, and is an open area of research. Currently, such approaches rely on simplifications of the elastic and hydrodynamic equations [17,18].

The second approach, referred to as the hydrodynamic approach, is to find the swimming velocity resulting from a given deforming motion (kinematics) of the body and/or fins. This is a purely hydrodynamic problem that excludes the need to solve elasticity equations. Typical theories in aquatic locomotion fall into this category. These theories assume that rhythmic or non-rhythmic deformation kinematics are given based on experimental motion capture data. The hydrodynamic approach does not ignore the net elastic effect. It is embedded in the body deformation kinematics, within the accuracy with which the 3D kinematic data was obtained. As more high-precision 3D kinematics data of aquatic organisms becomes available (e.g. [19,20]), the hydrodynamic approach can use it to obtain high-accuracy predictions of their flow field and swimming velocities. Here, we provide an algorithm for simulating free-swimming organisms using the hydrodynamic approach.

Some of the prior FRS studies involve simulating the flow around deforming organisms at a *specified* constant swimming velocity (e.g. [21–25]). These simulations provide valuable insight into the fluid mechanics of aquatic locomotion, but are limited by the assumption of constant *specified* swimming velocity. Computations based on this assumption miss two crucial phenomena occurring in real free-swimming organisms. First, the swimming velocity is seldom constant, but almost always oscillatory with a constant mean value in the steady swimming state. Second, these temporal changes in the swimming velocity are likely to affect the flow characteristics around the body both qualitatively and quantitatively. To include these details, FRS should also solve for the swimming velocity instead of taking it is as an input. This will account for the two-way coupling between the flow field around the body and its swimming velocity.

There are FRS studies that present simulations of self-propelling organisms, i.e. they do not specify the swimming velocity but obtain it as a solution. These studies include a self-propelling eel [27,11], a flagellar structure with a head [28], and a copepod [23]. In these approaches, typically, the flow field is advanced first and then additional equations of motion for the body are solved based on the hydrodynamic force on the body. This explicit coupling can cause numerical instability due to the non-linear nature of the fluid dynamics equations and the angular momentum equation for the body [29].

Kern and Koumoutsakos [11] recently performed FRS of self-propelling eels with detailed body geometry and reproduced flow features observed in experiments of Tytell and Lauder [10]. In their work the geometric detail was made possible using a body-fitted grid. A body-fitted grid has to be regenerated periodically in order to conform to the changing configuration of the eel body. Body-fitted grid methods are difficult to apply to problems with arbitrarily shaped complex objects and multibody configurations. This is because in these cases, grid regeneration can significantly add to the computational cost. Additionally, fluid solvers are based on either unstructured or curvilinear structured grids, both of which can be computationally