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A novel intuitionistic fuzzy C means clustering algorithm and its application to medical images

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ABSTRACT

This paper presents a novel intuitionistic fuzzy C means clustering method using intuitionistic fuzzy set theory. The intuitionistic fuzzy set theory considers another uncertainty parameter which is the hesitation degree that arises while defining the membership function and thus the cluster centers may converge to a desirable location than the cluster centers obtained using fuzzy C means algorithm. Also a new objective function which is the intuitionistic fuzzy entropy is incorporated in the conventional fuzzy C means clustering algorithm. This is done to maximize the good points in the class. This clustering method is used in clustering different regions of the CT scan brain images and these may be used to identify the abnormalities in the brain. Experimental results show the effectiveness of the proposed method in contrast to conventional fuzzy C means algorithms and also type II fuzzy algorithm.

1. Introduction

Segmentation is a key step towards image analysis in various image processing application such as object recognition, pattern recognition, medical imaging. It can be stated as partitioning the image into different regions each having homogeneous features such as color, texture, and so on. This type of segmentation is called clustering which is very important in classifying different patterns in an image. Generally unsupervised method is used to segment the image. Clustering may be crisp or fuzzy. But images are considered fuzzy due to the uncertainty present in terms of vagueness in class definitions, regions/boundaries, imprecise gray levels and so fuzzy clustering is the most widely used where an element may have partial membership grades in several fuzzy clusters.

While discussing the uncertainty, another uncertainty arises and this is the hesitation in defining the membership function of the pixels of an image. Since the membership degrees are imprecise and it varies on person's choice, there is some kind of hesitation present which arises from the lack of precise knowledge in defining the membership function. This idea leads to an introduction of another higher order fuzzy set called intuitionistic fuzzy set introduced by Atanassov's in 1983 [1]. It takes into account the membership degree as well as the non-membership degree. In an ordinary fuzzy set, the non-membership degree is the complement of the membership degree, but in intuitionistic fuzzy set the non-membership degree is less than or equal to the complement of the membership degree due to the hesitation degree.

The first fuzzy method to segment the regions of an image is the fuzzy C means clustering (FCM), introduced by Bezdek et al. [4]. The algorithm requires a priori definition of number of classes that will partition the image. It classifies the set of data points $X=(x_1, x_2, x_3,\ldots, x_n)$ into c homogeneous groups or clusters represented as fuzzy sets, $F=(F_1, F_2, F_3,\ldots, F_c)$. The objective is to obtain a 'c' partition by minimizing the criterion function:

$$J_m(U, V : X) = \sum_{i=1}^{c}\sum_{k=1}^{n}\mu^m{}_{ik}d^2(x_k, v_i)$$

where c is the number of clusters and μ_{ij} is the membership of the data x_j to the fuzzy cluster F_i. m is user defined and generally it is taken as 2. Euclidean distance or any distance measure is used to find the distance between the cluster center and the data points.

Many studies are reported in the literature to improve the fuzzy clustering algorithm. Ferahta et al. [8] had given the idea of optimal classes where the number of classes is not predefined. They introduced another criterion function that is the Shannon's entropy to maximize the good points in the class. Iyer et al. [9] used 21 features in FCM to cluster mammogram images. Rhee and Hwang [13] proposed Type 2 fuzzy clustering. Type 2 fuzzy set is the fuzziness in a fuzzy set. In this algorithm, the membership value of each pattern in the image is extended as Type 2 fuzzy membership by assigning membership grades (triangular membership function) to Type 1 fuzzy membership. The membership values for the Type 2

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membership are obtained as:

$$a_{ik} = u_{ik} - \frac{1-u_{ik}}{2}$$

where a_{ik} and u_{ik} are the Type 2 and Type 1 fuzzy membership respectively. The cluster centers are updated accordingly using conventional FCM taking into account the new Type 2 fuzzy membership.

Few work on clustering is reported in the literature on intuitionistic fuzzy sets. Studies on intuitionistic fuzzy set are done by Atanassov [3] on theory, and application on intuitionistic fuzzy sets. Yager [18] suggested some aspects of intuitionistic fuzzy sets. Khatibi and Montazar [11] classified the intestinal bacteria such as Salmonella and Shiegella using fuzzy and intuitionistic fuzzy set theory that causes typhoid and dysentery. In their work, uncertainty in the similarity degrees using five types of fuzzy sets and intuitionistic fuzzy sets are studied to develop an intelligent approach to classify the bacteria. Zhang and Chen [20] suggested a clustering procedure where an intuitionistic fuzzy similarity matrix is transformed to interval valued fuzzy matrix. From the similarity degrees between two intuitionistic fuzzy sets, intuitionistic fuzzy similarity matrix is created which is transformed to intuitionistic fuzzy equivalence matrix. Then, a clustering technique that uses intuitionistic fuzzy equivalence matrix is suggested based on the λ-cutting matrix of the intuitionistic fuzzy equivalence matrix. Xu et al. [15] suggested clustering algorithm for intuitionistic fuzzy sets based on the concept of association matrix and equivalent association matrix. They calculated the association coefficients in a different way where hesitation degree is taken into account and constructed association matrix and then to equivalent association matrix. Clustering is then done based on the λ-cutting matrix of the equivalent association equivalence matrix.

This paper proposes a novel intuitionistic fuzzy C means clustering algorithm using intuitionistic fuzzy set theory. This algorithm incorporates another uncertainty which is the hesitation degree that arises while defining the membership function. In the construction of the algorithm, the following steps required are as follows:

[1] Creating intuitionistic fuzzy set using Yager type intuitionistic fuzzy generator.
[2] Modifying the objective criterion.
[3] Updating the cluster center using intuitionistic fuzzy set.
[4] Incorporating an objective function i.e. the intuitionistic fuzzy entropy in the criterion function.

As there is no work in the literature on intuitionistic fuzzy C means clustering, so to show its efficacy, experiments are performed on CT scan brain images.

The paper is organized as follows. Section 2 outlines the preliminaries on the intuitionistic fuzzy set. Section 3 suggests intuitionistic fuzzy entropy of a intuitionistic fuzzy set. Section 4 discusses on the construction of intuitionistic fuzzy set using Yager type intuitionistic fuzzy generator. Section 5 presents the modified intuitionistic fuzzy C means algorithm along with a new criterion function. Section 6 displays the results and discussion and finally the conclusion is drawn in Section 7.

2. Preliminaries

Fuzzy set is generated only with membership function $\mu(x), x \in X$ but in Atanassov's [2] intuitionistic fuzzy set (IFS) both membership $\mu(x)$ and non-membership $\nu(x)$ functions are considered.

An intuitionistic fuzzy set A in X, is written as:

$$A = \{x, \mu_A(x), \nu_A(x) | x \in X\}$$

where $\mu_A(x) \to [0,1]$, $\nu_A(x) \to [0,1]$ are the membership and non-membership degrees of an element x in the set A with the condition

$$0 \le \mu_A(x) + \nu_A(x) \le 1$$

when $\nu_A(x) = 1 - \mu_A(x)$ for every x in set A, then the set A becomes a fuzzy set.

For all intuitionistic fuzzy sets, Atanassov also indicated a hesitation degree, $\pi_A(x)$, which arises due to lack of knowledge in defining the membership degree of each element x in set A and is given by:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \quad (1)$$

Obviously, $0 \le \pi_A(x) \le 1$.

Due to the hesitation degree, the membership values lie in the interval

$$[\mu_A(x), \mu_A(x) + \pi_A(x)].$$

3. Construction of intuitionistic fuzzy entropy

A new objective function i.e. intuitionistic fuzzy entropy is introduced in the clustering. So, initially the idea of fuzzy entropy is given. It is a measure of fuzziness in a fuzzy set. Zadeh [19] first introduced the idea of fuzzy entropy in 1969. Kaufmann [10] used distance measure to define fuzzy entropy whereas Yager [17] defined entropy as the distance from a fuzzy set and its complement. Similarly in the case IFS, intuitionistic fuzzy entropy gives the amount of vagueness or ambiguity in a set. Many authors defined intuitionistic fuzzy entropy in different manner. Szmidt and Kacpryzk [14] defined entropy in terms of non-probabilistic type of entropy. Burillo and Bustince [5] defined entropy in terms of the degree of intuitionism of an intuitionistic fuzzy set. The properties of intuitionistic fuzzy entropy [4] are:

A real function IFE:IFEs$(X) \to R^+$ is called intuitionistic fuzzy entropy (IFE) on IFEs(X) if

[1] IFE$(A) = 0$, $\forall A \in$ FS(X).
[2] IFE(A) = Cardinal(X) = n, iff $\mu_A(x_i) = \nu_A(x_i) = 0 \forall x_i$.
[3] If the degree of membership and non-membership of each element decreases, their sum will decrease and fuzziness in a set decreases, thereby the hesitation degree will increase and thus IFE will increase.

If $A \Leftarrow B$, $\mu_A(x_i) \le \mu_B(x_i)$, $\nu_A(x_i) \le \nu_B(x_i)$, $\forall x_i \in X$, implies $\mu_A(x_i) + \nu_A(x_i) \le \mu_B(x_i) + \nu_B(x_i)$.

So, $\pi_A(x_i) \ge \pi_B(x_i)$.

Thus IFE$(A) \ge$ IFE(B).

Pal and Pal [12] analyzed the classical Shannon information theory and introduced exponential entropy. For a probability distribution, $p = p_1, p_2, \ldots, p_n$, the exponential entropy is defined as: $H = \sum_{i=1}^{n} p_i e^{(1-p_i)}$.

For intuitionistic fuzzy cases, if $\mu_A(x_i)$, $\nu_A(x_i)$, $\pi_A(x_i)$ are the membership, non-membership, and hesitation degrees of the elements of the set $X = \{x_1, x_2, \ldots, x_n\}$, then intuitionistic fuzzy entropy, IFE that denotes the degree of intuitionism in fuzzy set, may be given as:

$$\text{IFE}(A) = \sum_{i=1}^{n} \pi_A(x_i) e^{[1-\pi_A(x_i)]} \quad (2)$$

where $\pi_A(x_i) = 1 - (\mu_A(x_i) + \nu_A(x_i))$.

IFE in Eq. (2) satisfies the condition of intuitionistic fuzzy entropy [5] which is as follows:

(1) IFE$(A) = 0$ if $1 - (\mu_A(x_i) + \nu_A(x_i)) = 0$,
which implies $(\mu_A(x_i) + \nu_A(x_i)) = 1$, for all x_i and thus $A \in FS$.