Fast algorithm for multiplicative noise removal

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1. Introduction

Image denoising problem has been widely studied in the areas of image processing. Most of the literature deals with the additive noise model. But in practice, there are other types of noise such as multiplicative noise. It can also corrupt an image. In this paper, we are interested in the multiplicative noise removal problem. This problem can be expressed as follows: given a recorded image \( g \) and a noise \( v \):

\[
g = u v. \tag{1}\]

Here, \( \Omega \) denotes the image domain that is simplified a rectangle domain in usual. The images we considered are 2-dimensional matrices of size \( M \times N \). Without loss of generality, we can suppose that each value of \( u \) and \( v \) are positive in the noise model. Due to this degraded mechanism, nearly all the information of the original image may vanish when it is distorted by multiplicative noise. Therefore, it is important to remove multiplicative noise. The goal of restoration is to recover the true image \( u \) from the data \( g \). The problem of removing multiplicative noise occurs in many applications, such as synthetic aperture radar, ultrasound imaging and laser imaging, see [14].

In literature, various variational approaches devoted to multiplicative noise removal have been proposed. The early variational approach for multiplicative noise removal is the one by Rudin et al. [14] as used for instance in [6,9,12,16]. By using a maximum a posteriori (MAP) estimator, Aubert and Aujol [2] proposed a functional whose minimizer corresponds to the denoised image to be recovered. This functional is:

\[
E(u) = \int_{\Omega} |Du| + \lambda \int_{\Omega} \left( \log u + \frac{g}{u} \right) dx dy, \tag{2}\]

where \( \int_{\Omega} |Du| \) denotes the total variation of \( u \) and \( \lambda \) is a regularization parameter. In their method, they considered the Gamma noise with mean one. Though the functional they proposed is not convex, they still proved the existence of the minimizer, gave a sufficient condition ensuring uniqueness and showed that a comparison principle holds. They further gave some numerical examples illustrating the capability of their model.

As a result of the drawback of the function (2) that is not convex for all \( u \), the solution for the method in [2] is likely not the global optimal solution of (2). Therefore, the quality of the denoised image may be not good. In view of this, Shi and Osher [15] presented a convex model which adopts the fitting term in (2). They adopted inverse scale space flow as denoising technique. Moreover, Huang et al. [4] proposed a strictly convex objective functional for
multiplicative noise removal by modifying the model in [15]. They also incorporated another way of modified total variation regularization in the objective function to recover image edges. In their paper, they considered a new variable 
\[ z = \log u. \]

Thus, the second term in (2) can be reformulated as 
\[ \int_\Omega (z + ge^{-z}) dx dy. \]  

(3)

By using the new term in (3), the proposed unconstrained total variation denoising problem is given as follows:
\[ \min_{z,w} \int_\Omega (z + ge^{-z}) dx dy + z_1 \| z - w \|_L^2 + z_2 \int_\Omega |Dw|, \]

(4)

where \( z_1 \) and \( z_2 \) are positive regularization parameters. They developed an alternating minimization algorithm to find the minimizer of (4) efficiently, and also proved the convergence of the minimizing method.

For multiplicative noise removal problem, there exist some other methods besides variational approaches, such as local linear minimum mean square error approaches [8,10] and anisotropic diffusion methods [1,7,17]. They will not be addressed in this paper.

In this paper, we propose a strictly convex objective functional for multiplicative noise removal problem by using anisotropic total variation (ATV) and the fitting term in (2). We establish an alternating minimization algorithm to find the minimizer of such objective functional efficiently, and give the convergence result of the alternating minimization algorithm. The proposed algorithm is easy to implement. And the computational speed is more than the alternating minimization algorithm. The proposed algorithm is much more efficient, and give the convergence result of the anisotropic functional efficiently, and give the convergence result of the anisotropic total variation (ATV) and the fitting term in (2). We establish an alternating minimization algorithm to find the minimizer of the anisotropic total variation and give the convergence result of the anisotropic total variation method.

2. The ATV multiplicative denoising model

The aim of this section is to propose a strictly convex objective functional for denoising images corrupted by multiplicative noise. We incorporate anisotropic total variation and the fitting term in (2) in the objective functional to recover image edges efficiently. We start from the multiplicative noise model (1). In the following, we assume that \( g, u, v \) are samples of the random variables \( G, U, V \) and denote the probability density function of a random variable \( X \) by \( f_X \). Moreover, we also assume that the samples of noise on each pixel \( x \in \Omega \) are mutually independent and identically distributed (i.i.d.) with density function \( f_V \).

2.1. The proposed model

We suppose that the multiplicative noise in each pixel follows a Gamma distribution with mean one and with its probability density function given by:

\[ f_V(v) = \begin{cases} \frac{\Gamma(z)}{\Gamma(z) L^z} e^{-\frac{v}{L}} , & v > 0, \\ 0 , & \leq 0, \end{cases} \]

where \( L > 0 \) is the number of looks (in general, \( L \) is a positive integer) and \( \Gamma(\cdot) \) is a Gamma function.

According to the maximum a posteriori estimation, the restored image \( \hat{u} \) can be computed by

\[ \hat{u} = \arg \max_u f_{U|G}(u|g). \]

Applying Baye's rule, it becomes

\[ \hat{u} = \arg \max_u \frac{f_{G|U}(g|u) f_U(u)}{f_G(g)}. \]

(5)

By using Proposition 3.1 in [2], we get:

\[ f_{G|U}(g|u) = f_V \left( \frac{g}{u} \right) \frac{1}{u} = \frac{1}{u} \frac{g^{z-1}}{\Gamma(z)} e^{-\frac{g}{u}}. \]

(6)

Taking the logarithm transformation into account, we assume that the image prior \( f_U(u) \) as follows:

\[ f_U(u) = f_{U|W}(u|w) f_W(w), \]

with

\[ f_{U|W}(u|w) \propto \exp(-z_1 \log u - \| u \|_L^2), \]

\[ f_W(w) \propto \exp(-z_2 (\| w \|_{L^1} + \| w \|_{L^2})). \]

where \( z_1 \) and \( z_2 \) are two positive constants. Herein, we suppose that the difference between \( \log u \) and \( w \) follows a Gaussian distribution and \( w \) obeys an anisotropic total variation prior. Therefore, we have

\[ \hat{u} = \arg \min_u \left( -\log f_{G|U}(g|u) \right) \]
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