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Fast algorithm for multiplicative noise removal ☆

Baoli Shi [a], Lihong Huang [a,c,*], Zhi-Feng Pang [b]

[a] College of Mathematics and Econometrics, Hunan University, Changsha, Hunan 410082, PR China
[b] College of Mathematics and Information Science, Henan University, Kaifeng, Henan 475004, PR China
[c] Hunan Women's University, Changsha, Hunan 410000, PR China

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ABSTRACT

In this work, we consider a variational restoration model for multiplicative noise removal problem. By using a maximum a posteriori estimator, we propose a strictly convex objective functional whose minimizer corresponds to the denoised image we want to recover. We incorporate the anisotropic total variation regularization in the objective functional in order to preserve the edges well. A fast alternating minimization algorithm is established to find the minimizer of the objective functional efficiently. We also give the convergence of this minimization algorithm. A broad range of numerical results are given to prove the effectiveness of our proposed model.

1. Introduction

Image denoising problem has been widely studied in the areas of image processing. Most of the literature deals with the additive noise model. But in practice, there are other types of noise such as multiplicative noise. It can also corrupt an image. In this paper, we are interested in the multiplicative noise removal problem. This problem can be expressed as follows: given a recorded image $g : \Omega \subset R^2 \to R$, which is the multiplication of an original image u and a noise v:

$$g = uv. \tag{1}$$

Here, Ω denotes the image domain that is simplified a rectangle domain in usual. The images we considered are 2-dimensional matrices of size $M \times N$. Without loss of generality, we can suppose that each value of u and v are positive in the noise model. Due to this degraded mechanism, nearly all the information of the original image may vanish when it is distorted by multiplicative noise. Therefore, it is important to remove multiplicative noise. The goal of restoration is to recover the true image u from the data g. The problem of removing multiplicative noise occurs in many applications, such as synthetic aperture radar, ultrasound imaging and laser imaging, see [14].

In literature, various variational approaches devoted to multiplicative noise removal have been proposed. The early variational approach for multiplicative noise removal is the one by Rudin et al. [14] as used for instance in [6,9,12,16]. By using a maximum a posteriori (MAP) estimator, Aubert and Aujol [2] proposed a functional whose minimizer corresponds to the denoised image to be recovered. This functional is:

$$E(u) = \int_\Omega |Du| + \lambda \int_\Omega \left(\log u + \frac{g}{u}\right) \mathrm{d}x\, \mathrm{d}y, \tag{2}$$

where $\int_\Omega |Du|$ denotes the total variation of u and λ is a regularization parameter. In their method, they considered the Gamma noise with mean one. Though the functional they proposed is not convex, they still proved the existence of the minimizer, gave a sufficient condition ensuring uniqueness and showed that a comparison principle holds. They further gave some numerical examples illustrating the capability of their model.

As a result of the drawback of the function (2) that is not convex for all u, the solution for the method in [2] is likely not the global optimal solution of (2). Therefore, the quality of the denoised image may be not good. In view of this, Shi and Osher [15] presented a convex model which adopts the fitting term in (2). They adopted inverse scale space flow as denoising technique. Moreover, Huang et al. [4] proposed a strictly convex objective functional for

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* Corresponding author at: College of Mathematics and Econometrics, Hunan University, Changsha, Hunan 410082, PR China. Fax: +86 731 88823938.

E-mail addresses: shibaoli1983@163.com (B. Shi), lhhuang@hnu.edu.cn (L. Huang), zhifengpang@163.com (Z.-F. Pang).

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multiplicative noise removal by modifying the model in [15]. They also incorporated another way of modified total variation regularization in the objective function to recover image edges. In their paper, they considered a new variable

$$z = \log u.$$

Thus, the second term in (2) can be reformulated as

$$\int_\Omega (z + ge^{-z})\mathrm{d}x\,\mathrm{d}y. \tag{3}$$

By using the new term in (3), the proposed unconstrained total variation denoising problem is given as follows:

$$\min_{z,w} \int_\Omega (z + ge^{-z})\mathrm{d}x\,\mathrm{d}y + \alpha_1\|z - w\|_{L^2}^2 + \alpha_2\int_\Omega |Dw|, \tag{4}$$

where α_1 and α_2 are positive regularization parameters. They developed an alternating minimization algorithm to find the minimizer of (4) efficiently, and also proved the convergence of the minimizing method.

For multiplicative noise removal problem, there exist some other methods besides variational approaches, such as local linear minimum mean square error approaches [8,10] and anisotropic diffusion methods [1,7,17]. They will not be addressed in this paper.

In this paper, we propose a strictly convex objective functional for multiplicative noise removal problem by using anisotropic total variation (ATV) and the fitting term in (2). We establish an alternating minimization algorithm to find the minimizer of such objective functional efficiently, and give the convergence result of the alternating minimization algorithm. The proposed algorithm is easy to implement. And the computational speed is more than six times of the method in [4]. The quality of restored images by our proposed method is quite well. This can be seen in our experimental results.

This paper is organized as follows: in Section 2, we recommend our proposed model and the alternating minimizing algorithm. In Section 3, we give the convergence result of the proposed method. In Section 4, numerical results are given to show the effectiveness of our method. In Section 5, we have a conclusion.

2. The ATV multiplicative denoising model

The aim of this section is to propose a strictly convex objective functional for denoising images corrupted by multiplicative noise. We incorporate anisotropic total variation and the fitting term in (2) in the objective functional to recover image edges efficiently. We start from the multiplicative noise model (1). In the following, we assume that g, u, v are samples of the random variables G, U, V, and denote the probability density function of a random variable X by f_X. Moreover, we also assume that the samples of noise on each pixel $x \in \Omega$ are mutually independent and identically distributed (i.i.d.) with density function f_V.

2.1. The proposed model

We suppose that the multiplicative noise in each pixel follows a Gamma distribution with mean one and with its probability density function given by:

$$f_V(v) = \begin{cases} \frac{L^L v^{L-1}}{\Gamma(L)} e^{-Lv}, & v > 0, \\ 0, & \leqslant 0, \end{cases}$$

where $L > 0$ is the number of looks (in general, L is a positive integer) and $\Gamma(\cdot)$ is a Gamma function.

According to the maximum a posteriori estimation, the restored image $\hat{u}$ can be computed by

$$\hat{u} = \arg\max_u f_{U|G}(u|g).$$

Applying Baye's rule, it becomes

$$\hat{u} = \arg\max_u \frac{f_{G|U}(g|u)f_U(u)}{f_G(g)}. \tag{5}$$

By using Proposition 3.1 in [2], we get:

$$f_{G|U}(g|u) = f_V\left(\frac{g}{u}\right)\frac{1}{u} = \frac{L^L g^{L-1}}{u^L\Gamma(L)} e^{-\frac{Lg}{u}}. \tag{6}$$

Taking the logarithm transformation into account, we assume that the image prior $f_U(u)$ as follows:

$$f_U(u) = f_{U|W}(u|w)f_W(w),$$

with

$$f_{U|W}(u|w) \propto \exp(-\alpha_1\|\log u - w\|_{L^2}^2),$$
$$f_W(w) \propto \exp(-\alpha_2(\|w_x\|_{L^1} + \|w_y\|_{L^1})),$$

where α_1 and α_2 are two positive constants. Herein, we suppose that the difference between $\log u$ and w obeys a Gaussian distribution and w obeys a anisotropic total variation prior. Therefore, we have

$$f_U(u) \propto \exp(-\alpha_1\|\log u - w\|_{L^2}^2)\exp(-\alpha_2(\|w_x\|_{L^1} + \|w_y\|_{L^1})). \tag{7}$$

Since $f_G(g)$ is a constant, (5) can be reformulated as

$$\hat{u} = \arg\max_u f_{G|U}(g|u)f_{U|W}(u|w)f_W(w). \tag{8}$$

For the above problem, we take logarithm transformation in order to transform multiplication to summation. Therefore, (8) can be rewritten as the following problem:

$$\hat{u} = \arg\min_u \left(-\log f_{G|U}(g|u) - \log f_{U|W}(u|w) - \log f_W(w)\right). \tag{9}$$

Using (6)–(8), we see that (9) amounts to:

$$\hat{u} = \arg\min_u \sum_{(x,y)\in\Omega}\left(L\left(\log u(x,y) + \frac{g(x,y)}{u(x,y)}\right) + \alpha_1\|\log u(x,y) - w\|_{L^2}^2 + \alpha_2\left(\|w_x\|_{L^1} + \|w_y\|_{L^1}\right)\right). \tag{10}$$

Based on the previous computation, we propose the following minimization problem by considering a new variable $z = \log u$:

$$\min_{z,w}\int_\Omega (z + ge^{-z})\mathrm{d}x\mathrm{d}y + \beta_1\|z - w\|_{L^2}^2 + \beta_2(\|w_x\|_{L^1} + \|w_y\|_{L^1}). \tag{11}$$

It is straightforward to check that (11) is equivalent to (9).

We will explain the proposed model. For the transformation $z = \log u$, it is obvious that when u includes an edge, z also contains an edge at the same location, i.e. the logarithm transformation preserves image edges. In view of this, we can consider z as an image in the logarithm domain. We note that the argument u should be positive in the objective functional (2). It will affect the quality of restoration image. However, in our model (11), the argument z can be any real number, and the corresponding $u = e^z$ is still positive. The main advantage of the first term in (11) is that its second derivative with respect to z is equal to ge^{-z}, which is always greater than zero. Therefore, this term is strictly convex for all z. We incorporate the fitting term $\|z - w\|_{L^2}^2$ in (11). The parameter β_1 measures the trade off between an image obtained by a maximum a posteriori estimation and a anisotropic total variation denoised image w. The parameter β_2 is used to measure the amount of regularization to a denoising image w. The anisotropic total variation can preserve the edges well because its diffusion is adapted to the direction of the local image features. It means that the diffusion strength along the direction which is vertical to the direction of local features is smaller than it along the direction of the local features.