Neurocomputing 119 (2013) 131–138

Contents lists available at SciVerse ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

A novel compound regularization and fast algorithm for compressive sensing deconvolution

Liang Xiao [a,b,*], Jun Shao [a,b], Lili Huang [a,b], Zhihui Wei [a,b]

[a] School of Computer Science, Nanjing University of Science and Technology, Nanjing 210094, China
[b] Jiangsu Province Key LAB of Spectral Imaging and Intelligent Sensing, Nanjing 210094, China

ARTICLE INFO

Available online 2 January 2013

Keywords:
Compressive sensing
Deconvolution from incomplete measurements
Compound regularization
Variable splitting

ABSTRACT

Compressive Sensing Deconvolution (CS-Deconvolution) is a new challenge problem encountered in a wide variety of image processing fields. Since CS is more efficient for sparse signals, in our scheme, the input image is firstly sparse represented by curvelet frame system; then the curvelet coefficients are encoded by a structurally random matrix based CS sampling technique. In order to improve the CS-deconvolution performance, a compound variational regularization model, which combined total variation and curvelet-based sparsity prior, is proposed to recovery blurred image from compressive measurements. Given the compressive measurements, we propose a novel fast algorithm using variable-splitting and Dual Douglas–Rachford operator splitting methods to produce high quality deblurred results. Our method considerably improves the visual quality of CS reconstruction for the given number of random measurements and reduces the decoding computational complexity, compared to the existing CS-deconvolution methods.

1. Introduction

Deconvolution is a classical signal/image processing problem and has wide applications in physical, optical, medical, and astronomical applications. Deconvolution from compressed measurements becomes necessary when we wish a crisp deblurred image for viewing or further processing [1]. One always expects if we can obtain high-resolution data from highly incomplete measurements, while saving the cost of measurements. However, in the traditional sensing (TS) framework, an image with limited bandwidth can only be reconstructed from the enough measurements by Shannon/Nyquist sampling theorem: the sampling rate must be at least twice the maximum frequency of signals. It seems we cannot reconstruct high-resolution data from highly incomplete measurements. Recently, a theory called compressed sensing (CS) or compressive sampling [2] makes us to deal with this problem. The theory illustrates that a sparse unknown signal can be recovered from a small number of random measurements by sparsity-promoting non-linear recovery algorithms. Different from TS, data acquisition by CS depends on its sparsity rather than its bandwidth. Actually, most natural scenes are sparse in suitable transforms such as Fourier, wavelets and curvelets [3,4]. The basic idea of CS is that the signal or image sparsity provides a heuristic prior knowledge about the signal, thus we can guess the high-resolution data from the incomplete measurements.

The deconvolution process is a mathematically ill-posed problem. Traditional methods such as wiener filtering and Richardson–Lucy deconvolution were proposed decades ago, but are still widely used in many image restoration tasks nowadays because they are simple and efficient. However, these methods tend to suffer from unpleasant ringing artifacts that appear near strong edges [5]. In order to tackle the ill-posed nature of deconvolution, the total variation (TV) based approaches have been developed to cope with the deconvolution problem [6,7]. Sparsity prior based regularization is another way to improve the performance of deconvolution. In frame-based synthesis formulations [8,9], the unknown image is represented on a frame (e.g., of wavelets, curvelets, etc.) and the coefficients of this representation are estimated, under some regularizer. For example, Donoho in [10] gave the first discussion of wavelet thresholding in linear inverse problems and introduced the wavelet-vaguelette decomposition. Authors in [11] have recently proposed a hybrid approach coined ForWaRD deconvolution methods with sparsity promoting regularization over wavelet coefficients. However, the conventional wavelets do not perform well at representing line singularities because they ignore the geometric properties of objects and do not exploit the regularity of edge curves, which leads to oscillatory artifacts along the edges.

Compared with traditional deconvolution problem, CS-deconvolution has more challenges. This is because in the case of compressive sampling, the CS-deconvolution process will be more ill-conditioned. CS-deconvolution has prospective applications in

* Corresponding author at: School of Computer Science, Nanjing University of Science and Technology, Nanjing 210094, China. Tel.: +86 25 8431 5645.
E-mail address: xiaoliang@mail.njust.edu.cn (L. Xiao).

0925-2312/$ - see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.neucom.2012.01.049

some practical areas, such as compressive imaging, medical imaging, distributed Compressed Sensing, analog to information conversion and bio-sensing. Most of the recent papers study two problems of CS. One is to find the optimal sampling ensembles and study the methods for fast implementation of the CS ensembles. The other one is to develop fast and practical reconstruction algorithms to recover the signal and suppress the noise introduced by CS.

The remainder of this paper is organized as follows. Section 2 introduces basic models and related algorithms for CS and CS-deconvolution. In Section 3, combing total variation and curvelet-based sparsity prior, we first propose a compound variational regularization model to recovery blurred image from compressive measurements. To solve the optimizing problem, we propose a novel fast algorithm using the variable-splitting and Dual Douglas–Rachford operator splitting methods. In Section 4, some experiments will be conducted and their results together with relevant discussions will be reported. The conclusions are finally summarized in Section 5.

2. Basic models and related algorithms

2.1. Observing model

In this paper, we treat the image reconstruction problem from compressed measurements. The problem setup is described as follows [1]:

$$f = \Phi H u + n \tag{1}$$

Here u is an unknown original discrete signal of size $N \times 1$, H is a $N \times N$ lowpass linear blurring operator, n is measurement error or noise, f is the observed signal and $\Phi \in C^{K,N}, K \ll N$ is so-called CS measurement matrix. Because $K \ll N$, the recovery of the $N \times 1$ signal u from the observation f of dimension $K \times 1$ leads to an underdetermined system, which is an ill-posed problem. But CS theory tells us that a sparse unknown signal can be recovered from a small number of random measurements by sparsity-promoting non-linear recovery algorithms. Suppose the image u is sparse in a specific transform Ψ system, thus the problem can be rewritten as follows:

$$f = \Phi H \Psi^{-1} \alpha + n \tag{2}$$

where $\alpha = \Psi u$, α is the sparse representation coefficients.

In this paper we will use fast compressive sampling based on structurally random matrix (SRM) to get the compressive measurements. Structurally Random Matrices are first proposed in [12] as fast and highly efficient measurement operators for large-scale compressed sensing applications. Actually, we adopt a fast scrambling-based dimension reduction algorithm, where Φ takes the following form:

$$\Phi = \sqrt{N/K} DTR \tag{3}$$

In which R is a $N \times N$ uniform random permutation operator, T represents a $N \times N$ fast-computable unitary matrix created by FFT transform, and D is a sampling operator.

As illustrated in Fig. 1, the problem of image deconvolution from structurally random measurements is to design a recovery algorithm to obtain the high-resolution data from the highly incomplete measurements, while saving the cost of measurements. This problem is more challenge compared with the classical image deconvolution, since its ill-posed property not only comes from the blurring matrix and the noise effect, but also comes from the measurement matrix.

2.2. Problem formulation

Based on the sparse reconstruction theory, the recovery problem can be carried out by l_1-minimization:

$$\min_u \|f - \Phi H u\|_2^2 + \lambda \|\Psi u\|_{l_1} \tag{4}$$

or

$$\min_\alpha \|\alpha_0 - \Phi H \alpha\|_2^2 + \lambda \|\alpha\|_{l_1} \tag{5}$$

where the first term is the data fidelity term which measures the difference between the solution and the observation. The second term in the above minimization problem is a regularization term that controls the norm (or seminorm) of the solution. The regularization parameter $\lambda > 0$ provides a tradeoff between fidelity to the measurements and noise sensitivity.

Let us denote $\mathbf{A} = \Phi H$, and Ψ^{-1} contains a transform basis (i.e., multiplying by Ψ^{-1} corresponds to performing inverse transform). The vector $\alpha_0 = \Psi f$ contains the coefficients of the observed image. The underlying philosophy in dealing with the l_1 norm regularization criterion is that most images have a sparse representation in some specific transform domain. Eq. (2) is an ill-conditioned underdetermined problem and one cannot solve it directly to recover u. However, if $\mathbf{A}$ satisfies a property known as the restricted isometric property(RIP) [2], u can be recovered by solving the problem of Eq. (4) or Eq. (5).

The convex optimization problem (5) can be cast as a second order cone programming problem and thus could be solved via interior point methods. However, the problem is not only large scale but also involves dense matrix data, which often precludes the use and potential advantage of sophisticated interior point methods.

One of the most popular methods for solving problem (5) is in the class of iterative shrinkage-thresholding algorithms (ISTA), where each iteration involves matrix–vector multiplication involving A and A^T followed by a shrinkage soft threshold step; see, e.g., [13]. Specifically, the general step of ISTA is

$$\alpha^t = T_{\lambda_t}((u^{t-1} + 2tA^T(f - Au^{t-1})), \psi) \tag{6}$$

where t is an appropriate step size and $T_\lambda \alpha : R^n \to R^n$ is the shrinkage operator defined by

$$T_\lambda \alpha = (|\alpha| - \lambda)_+ \cdot \operatorname{sgn}(\alpha) \tag{7}$$

In the optimization literature, this algorithm can be traced back to the proximal forward backward iterative scheme introduced in the general framework of splitting methods; see [14] and the references therein for a very good introduction to this approach, including convergence results. From the point view of Bayesian inference, the l_1 norm based sparse coding method can be used to make predictions about uncertainty, and to perform model selection. Nonetheless, the l_1 sparse coding methods are widely accepted and used, such as for example in the field of machine learning and pattern classification [15,16].

When the iterative shrinkage-thresholding is done in curvelet domain, the algorithm is called Curvelet-IST. Under the Curvelet-IST framework, Jianwei Ma proposed that the deconvolution operator can be embedded into Eq. (6) to improve the reconstruction performance [1]:

$$\alpha^t = T_{\lambda_t}((u^{t-1} + 2tP^{-1}\Phi^T(f - \mathbf{A}u^{t-1})), \psi) \tag{8}$$

Thus, using different deblurring operator, we can obtain different ISTA-deconvolution algorithm. When Poisson singular integral (PSI) [17] deconvolution operator is chosen as the deblurring operator, we call Curvelet-IST-PSI algorithm. Similarly, when Fourier based regularization deblurring (FoRD) [10] is used, it will derive a new compressive sensing deblurring algorithm