Computer Networks 52 (2008) 3229–3247

Contents lists available at ScienceDirect

Computer Networks

journal homepage: www.elsevier.com/locate/comnet

A fast algorithm for computing minimum routing cost spanning trees

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ARTICLE INFO

Article history:
Received 18 November 2007
Received in revised form 28 May 2008
Accepted 17 August 2008
Available online 27 August 2008

Responsible Editor: V.R. Syrotiuk

Keywords:
Minimum Routing Cost Tree
Shortest Path Tree
Minimum Spanning Tree
Bridging
Routing

ABSTRACT

Communication networks have been developed based on two networking approaches: bridging and routing. The convergence to an all-Ethernet paradigm in Personal and Local Area Networks and the increasing heterogeneity found in these networks emphasizes the current and future applicability of bridging. When bridging is used, a single active spanning tree needs to be defined. A Minimum Routing Cost Tree is known to be the optimal spanning tree if the probability of communication between any pair of network nodes is the same. Given that its computation is a NP-hard problem, approximation algorithms have been proposed.

We propose a new approximation Minimum Routing Cost Tree algorithm. Our algorithm has time complexity lower than the fastest known approximation algorithm and provides a spanning tree with the same routing cost in practice. In addition, it represents a better solution than the current spanning tree algorithm used in bridged networks.

1. Introduction

Along with the increasing convergence to an all-IP paradigm, in recent years, we have been witnessing a convergence to what can be called an all-Ethernet paradigm, especially in Personal Area Networks (PANs) and Local Area Networks (LANs). In fact, standards such as IEEE 802.11 [1] and Bluetooth with the specification of the PAN profile [2], and the recent draft specification of the WiMedia Network (WiNet), that builds on the WiMedia UWB (Ultra Wide Band) radio platform [3,4], have been defined to appear in the upper layers of the OSI (Open Systems Interconnection) model as legacy Ethernet links. This paradigm eases the use of bridging in scenarios beyond the traditional Ethernet LANs. For instance, IEEE 802.1D bridges [5] are being used to interconnect IEEE 802.11 networks to backhaul Ethernet infrastructures, to create Bluetooth PANs, and to interconnect Bluetooth PANs to IEEE 802 networks [2]. Additionally, they have been proposed to interconnect UWB WiNet networks to IEEE 802 networks [4] and pointed out by upcoming standards, such as IEEE 802.11s [6], as a solution to interconnect Layer 2 mesh networks to other IEEE 802 networks. The use of bridging enables the creation of a single Layer 2 network from the point of view of the upper layers and hides from them the multiplicity of underlying links that may form a PAN/LAN. New PAN/LAN wired or wireless technologies can be smoothly integrated with existing technologies, without modifications to the protocol stack above the data link layer of the OSI model. Also, IP and its companion protocols, such as Dynamic Host Configuration Protocol (DHCP) [7] and Address Resolution Protocol (ARP) [8], are enabled to run transparently over multiple underlying links.

When bridging is used a single spanning tree needs to be defined as the active network topology. Several spanning trees can be computed from the graph that models the network topology, also known as the topology graph. In the topology graph, the vertices model the network nodes, the edges model the network links connecting the network nodes, and the weights assigned to the edges represent costs computed based on metrics, such as bandwidth and delay. A Minimum Routing Cost Tree (MRCT) is, by definition, the optimal spanning tree from the

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doi:10.1016/j.comnet.2008.08.013

standpoint of the routing cost and always represents a spanning tree closer to the optimal solution defined by the union of the Shortest Path Trees.

The need for a single active spanning tree has been often presented as one of the major disadvantages of bridging. However, for heterogeneous PANs/LANs, i.e., networks whose corresponding topology graphs have heterogeneous edge weights, this may not represent a significant disadvantage. In Mieghem et al. [9,10] have demonstrated that, as the networks become increasingly heterogeneous, the union of the Shortest Path Trees tends to converge to a single spanning tree, a Minimum Spanning Tree (MST), which also defines an approximate or the exact MRCT in such cases. Therefore, with the increasing heterogeneity found in PANs/LANs – consider, for example, an Ethernet network where 100 Mbit/s and 1 Gbit/s links may coexist in the same LAN, or a future PAN where IEEE 802.11 (54 Mbit/s), Bluetooth (3 Mbit/s), and UWB WiNet (480 Mbit/s) links may coexist, the use of bridging gains a new interest.

Regarding the computation of a spanning tree from a mesh network topology, the IEEE has specified the Rapid Spanning Tree Protocol (RSTP) included in the IEEE 802.1D standard [5]. The devices participating in the protocol elect a root node (called the root bridge) and each device computes the shortest path towards the elected root. The union of these paths represents the final spanning tree, a Shortest Path Tree (SPT) from the root node's perspective. Nonetheless, an arbitrary SPT is selected to define the active network topology, as a consequence of the arbitrary selection of the root node. In practice, the arbitrary SPT does not define a good approximation to an MRCT, in particular for heterogeneous networks, precisely where the use of bridging would be of greater interest.

The computation of an MRCT is known to be a NP-hard problem [11]. That fact has led to the development of approximation algorithms. The fastest known approximation algorithm has been proposed by Wu et. al [12] but it only applies to metric graphs, i.e., graphs whose edges weights obey to the triangle inequality. In practice, edge weights may not obey to this condition since it may frequently happen that the direct path between two adjacent nodes is longer than some indirect path between them. As such, the algorithm is not generically applicable. The fastest known generic approximation algorithm has been proposed by Wong [13]. *Wong's* algorithm specifies the computation of n Shortest Path Trees (SPTs), where n is the number of nodes in the input graph, and selects the tree with the lowest routing cost. Still, the mandatory computation of n SPTs and the calculation of the routing cost for each of them represent its main disadvantages. More recently, Grout [14] has proposed a greedy approximation MRCT algorithm, called *Add* algorithm, that provides good results for homogeneous graphs (i.e., graphs whose edges weights are equal) and lower time complexity than *Wong's* algorithm. Nonetheless, it does not work for non-homogeneous graphs. We herein propose a new approximation MRCT algorithm, called *Campos's* algorithm, that has lower time complexity than *Wong's* algorithm while providing a spanning tree with similar routing cost for both homogeneous and heterogeneous graphs in practice. Furthermore, *Campos's* algorithm provides results better than the current spanning tree algorithm used in bridged networks, namely for heterogeneous networks.

The motivation for the present work is three-fold. Firstly, the relevance that bridging has and is envisioned to have in the future within the PAN/LAN scope. Secondly, the shortcomings found in the spanning tree algorithm currently used by IEEE 802.1D bridges; in general, it does not represent a good approximation MRCT algorithm. Thirdly, the lack of an approximation MRCT algorithm that provides similar results to *Wong's* algorithm (in practice) and has lower time complexity.

The contributions of our work are the following. Firstly, we propose *Campos's* algorithm as a new approximation MRCT algorithm. To the best of our knowledge, it is the first approximation MRCT algorithm with time complexity lower than *Wong's* algorithm while offering similar routing costs in practical cases. Furthermore, it represents a different approach concerning improvements to the current spanning tree algorithm used by IEEE 802.1D bridges; previous proposals have focused more on the provisioning of loop freedom or on the improvement of the performance of the spanning tree protocol itself, and less on the computation of the spanning tree used as active topology. Secondly, we make a comparative analysis of multiple spanning tree algorithms against the current spanning tree algorithm used in IEEE 802.1D bridges by using simulations. Previous studies have focused more on asymptotical analysis or have considered a limited scope; for instance, in [14] Grout has compared his algorithm with MST algorithms but did not compare it with IEEE algorithm and/or *Wong's* algorithm. Thirdly, we conclude about the realistic scenarios where *Add* and the classical MST algorithms can be used as faster alternatives to *Wong's* algorithm for computing approximate MRCTs.

Based on the simulation results we conclude that: (1) *Campos's* algorithm does represent the current fastest approximation MRCT algorithm and does provide results similar to those obtained using *Wong's* algorithm for practical cases; (2) as the number of vertices in the current graph increases, *Campos's* algorithm performs better than *Add* algorithm, when sparse homogeneous graphs are considered; (3) *Campos's* algorithm gives lower routing costs than the spanning tree algorithm currently used by IEEE 802.1D bridges and may be used in current and new bridging oriented solutions to define the active spanning tree; (4) *Add* algorithm is a good approximation MRCT algorithm as far as homogeneous graphs are concerned, but it performs poorly for heterogeneous graphs; (5) MST algorithms tend to approximate the result provided by *Wong's* algorithm as the weights in the input graph become increasingly heterogeneous.

Along the paper we take three major assumptions. Firstly, we limit our analysis to networks with up to 50 nodes. We focus on small scale networks, such as PANs and LANs, as this represents the typical applicability domain for bridging. Secondly, while computing an approximate MRCT our goal is to find the approximate unconstrained MRCT. That is, we do not assume any constraints, such as, minimum node degree in the final spanning tree. Thirdly, we assume a random communication