Available online at www.sciencedirect.com
ScienceDirect
Signal Processing 87 (2007) 789–798
SIGNAL PROCESSING
www.elsevier.com/locate/sigpro

Fast algorithms for polynomial time frequency transform

Yongmei Wei, Guoan Bi*

Block S1, School of Electrical and Electronic Engineering, Nanyang Technological University, Nanyang Avenue, Singapore 639798, Singapore

Received 22 August 2005; received in revised form 26 May 2006; accepted 31 July 2006
Available online 1 September 2006

Abstract

The computation of polynomial time frequency transform (PTFT) is required for the maximum likelihood method to estimate the phase parameters of the polynomial-phase signals (PPSs). The transform can be computed by directly using the 1D fast Fourier transforms (FFT), which requires a prohibitive computational load for higher-order PPSs. By exploiting two properties of the PTFT, this paper presents a decimation-in-time fast algorithm to significantly reduce the computational complexity compared with that by only using 1D FFT. For example, the numbers of both complex multiplications and additions are reduced by a factor of $2^M \log_2 N$ for N-point $(M+1)$th-order PTFTs.

Keywords: FFT; Polynomial-phase signals; Polynomial time frequency transform

1. Introduction

The polynomial-phase signals (PPSs) are encountered in many applications [1–3], such as synthetic aperture radar (SAR) imaging [4,5] and mobile communications [6,7]. A single component $(M+1)$th-order PPS embedded in white Gaussian noise $w(n)$ has the form:

$$s(n) = A(n)e^{j2\pi\sum_{m=0}^{M+1} a_m n^m} + w(n),\quad n = 0, 1, \ldots, N-1, \tag{1}$$

where the PPS has an amplitude $A(n)$ which is either assumed to be constant or a real-valued stationary Gaussian process with any structure, and parameters a_m, $m = 0, 1, \ldots, M, M+1$, are associated with the polynomial phase. For signal analysis, estimation of the parameters, a_m, from the PPSs has been extensively studied [1–3,8–12] in the past years. In the parametric category, there exist maximum likelihood (ML) methods [1,9,10,13,14] and sub-optimal ML estimation methods such as those based on high-order ambiguity function (HAF) [3] and its variations [2,8,15]. This paper reports fast algorithms for the polynomial time frequency transform (PTFT) to support the ML estimation t hat has the advantages of better statistical performances for high-order PPS even under low input signal-to-noise ratio (SNR) compared with those sub-optimal methods [1,9,10,15]. These advantages are important for applications where the input SNR is limited by the power of the transmitter or higher PPS modeling is required. For example, long

*Corresponding author. Tel.: +65 67904823; fax: +65 67933318.
E-mail address: egbi@ntu.edu.sg (G. Bi).

0165-1684/$ - see front matter © 2006 Elsevier B.V. All rights reserved.
doi:10.1016/j.sigpro.2006.07.010

synthetic aperture, which means long observation interval, in SAR is required to obtain a high resolution to recognize targets moving on the ground hidden by foliage [8,16]. In this case, higher-order PPS is required because the second-order PPS is not appropriate for the signals to be observed within long intervals [8,16]. In recent years, ML methods attract much attention and several ML methods [1,9,10,13–15] were reported for estimating the parameters of PPSs in the literature. The general form of the transform for ML estimation is derived in Appendix A in [1]. In this paper, we call the transform as PTFT [17]. Although the algorithms of fast Fourier transform (FFT) are available for use, the PTFT requires a huge computational complexity to deal with multidimensional calculation. Therefore, finding a fast algorithm supporting the $(M+1)$th-order PTFT becomes a critical issue for any practical applications. Fast quadratic phase transform [13] was proposed as an efficient algorithm for the second-order PTFT to reduce the multiplicative complexity by a factor of $\log_2 N$ compared with that needed by directly using the one-dimensional (1D) FFTs [18,19]. Recently, the work in [13] was extended and a fast algorithm for the third-order PTFT was reported in [17]. To the best knowledge of the authors, there has not been a fast algorithm to support arbitrary order PTFT based on the decimation-in-time (DIT) decomposition technique in the literature. This paper generalizes the algorithms in [13,17] and presents a general fast algorithm for the PTFTs of any order by exploiting the intrinsic properties of the PTFT to remove the redundant computation. For example, the proposed fast algorithm reduces the computational load by a factor of $2^M \log_2 N$ compared to the algorithm that directly uses the 1D FFTs.

This paper is organized as follows. In Section 2, the definition of the PTFT is given and the quasi-periodic property is shown to redefine the searching space of PTFT. Section 3 briefly reviews the process of the radix-2 FFT algorithm that is to be used to derive the key property for the proposed fast algorithm. Based on the quasi-periodic property and the key property of the PTFT, Section 4 presents the details of the fast algorithm. In Section 5, computational complexity of the proposed algorithm is discussed and compared with other reported ones in the literature. Finally, conclusion is given in Section 6.

2. Polynomial time frequency transform

The maximum likelihood estimation (MLE) of $\mathbf{a} = \{a_1, \ldots, a_{M+1}\}$ for the PPSs defined in (1) is expressed as

$$\{\hat{a}_1, \ldots, \hat{a}_{M+1}\} = \arg\max_{\theta} \left| \sum_{n=0}^{N-1} s^2(n) \mathrm{e}^{-j2\pi \sum_{m=0}^{M+1} \theta_m n^m} \right|^2, \tag{2}$$

if $A(n)$ is constant [1,9,10] and

$$\{\hat{a}_1, \ldots, \hat{a}_{M+1}\} = \frac{1}{2} \arg\max_{\theta} \left| \sum_{n=0}^{N-1} s(n) \mathrm{e}^{-j2\pi \sum_{m=0}^{M+1} \theta_m n^m} \right|^2, \tag{3}$$

if $A(n)$ is a real-valued stationary mixing Gaussian process with any structure [1]. In the above equations, $\mathbf{\Theta} = \{\theta_1, \ldots, \theta_{M+1}\}, (M+1)$ is the number of parameters and N is the total number of the samples from the analyzed signal. It is worth mentioning that (3) is the nonlinear least-square estimation of $\mathbf{a}$ when $A(n)$ is not Gaussian. Since the main focus in this paper is the derivation of fast algorithms for the PTFTs, the estimation procedures, which can be found in [1,3], are omitted.

2.1. Definition of PTFT

We define the $(M+1)$th-order PTFT, from which the MLE of the parameters is obtained, as

$$\mathrm{PTFT}_x^{M+1}(k, \mathbf{l}) = \frac{1}{N} \sum_{n=0}^{N-1} x(n) \mathrm{e}^{-j2\pi\phi_1(k,\mathbf{l},n)}, \tag{4}$$

where $x(n) = s^2(n)$ for time-varying $A(n)$, $x(n) = s(n)$ for constant $A(n)$, and

$$\begin{aligned} \phi_1(k, \mathbf{l}, n) = (k/N)n + (l_1/N_1)n^2 + (l_2/N_2)n^3 \\ + \cdots + (l_M/N_M)n^{M+1}, \end{aligned}$$

$$\mathbf{l} = (l_1, l_2, \ldots, l_M); \quad l_i = 0, \ldots, N_i - 1; \quad k = 0, \ldots, N-1. \tag{5}$$

Since it is obvious from (1) that the period of a_i is one, the analyzed range of a_i is [0,1). The candidates for each l_i are sampled in the range of [0,1). For each l_i in (5), N_i is the total number of values of l_i and N is assumed to be 2^q with a positive integer q. To achieve a satisfactory accuracy of the parameter estimation, it is necessary that $N_j \geqslant N_i \geqslant N$ for $j > i$, which is similar to the criterion in [13]. In the rest of