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Conjectural variation based bidding strategy in spot markets: fundamentals and comparison with classical game theoretical bidding strategies

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Abstract

In this paper, the concept of conjectural variation (CV) and its applications in electricity spot markets are introduced. The conjecture of a firm is defined as its belief or expectation of how its rivals will react to the change of its output. CV based bidding strategy (CVBS) method can help generation firms to improve their strategic bidding and maximize their profits in real electricity spot markets with imperfect information. In real applications, a firm using CVBS will integrate its rivals into one fictitious competitor and estimate its generation and reaction to the firm's change of output so that an optimal decision can be made accordingly. It is shown that classical game theoretic bidding strategies (GTBS) are special cases of CVBS families, and the system equilibrium reached via CVBS is a Nash equilibrium. Computer test results support the analytic conclusions very well.

Keywords: Electricity market; Bidding strategy; Conjecture variation; Game theory

1. Introduction

In recent years, competitions have been introduced to power industry in order to improve social welfare and market efficiency. However due to high barriers for new entrants to enter electricity markets caused by long construction period of power plants and huge amount of capital investment, the electricity market appears closer to an oligopolistic market with fewer power suppliers. Therefore, each generation firm will rationally conduct strategic bidding to maximize its own profit. Three main approaches, i.e. market clearing price (MCP) forecasting, rivals' bidding curve modeling and game based rivals' strategic behavior simulating [1–6] are often used for generation firm strategic bidding. Among them the game theory based method is most suitable to analyze the behavior in an oligopolistic electricity market. However, its application is limited by the requirement of common knowledge on all firms' actual production costs.

Recently, the conjectural variation (CV) based method is proposed to estimate the strategic behavior in game-theoretic context in terms of imperfect information available in actual electricity market [7,8]. The concept of conjectural variation was brought forward by Bowley in 1924, but named as 'conjectural variation' by Frisch in 1933 [9,10]. In conjectural variation models each firm in an oligopolitic market rationally maximizes its own profit while taking account of reactions of its competitors. The method can easily model a market with different players, such as leaders, followers, and price takers. The consistency of conjecture has been discussed by economists in [10–12] to show the dynamic characteristics of CV. The CV method is broadly applied in transportation decision, investment decision and other economic environments [13,14]. In power systems, the conjectural supply function is proposed to simulate the electricity market of Spain effectively [7], and in [8] it is applied to simulate the E&W market using linear dc network. However, the previous work on CV is

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still remaining in a static framework and requiring cost functions to estimate the conjectural parameters [8], which might make the conjecture of each firm different from the actual response of its rivals and therefore lead to less firm profits.

In this paper, the concept of CV and its applications in electricity spot markets are introduced. The conjecture of a firm is defined as its belief or expectation of how its rivals will react to the change of its output. CV based bidding strategy (CVBS) method can help generation firms to improve their strategic bidding and maximize their profits in actual electricity spot markets with imperfect information. In real markets with multiple players, a firm using CVBS will integrate its rivals' responses into one pseudo-competitor's response and use only available imperfect information announced in the market to make optimal decision accordingly. It is shown that classical game theoretical bidding strategies (GTBS) are special cases of CVBS families, and the system equilibrium reached via CVBS is a Nash equilibrium. Computer test results support the analytic conclusions very well.

The paper is arranged as follows. Section 2 presents math model of CV based bidding. Section 3 gives the application of CV based bidding strategy in real N-player power markets. Comparison of CVBS with classical GTBS approaches is presented in Section 4, together with the proof that if CVBS approach converges, it will converge to a Nash equilibrium. Section 5 is a computer test with conclusions drawn in Section 6.

2. Concept of conjectural variation

It is known from industrial organization theory [16] that *the CV of a firm is its belief or expectation on its rivals' reaction to its output changes.*

Let's consider a duopoly market with homogeneous product. Assume the market price (p) is

$$p = p(q_1,\ q_2) = p(Q) \tag{1}$$

where q_i ($i = 1, 2$) is the product quantity of firm i, $Q = q_1 + q_2$. For firm i, its objective is to maximize its profit via optimizing q_i, i.e.

$$\max_{q_i}\ \pi_i = pq_i - Cost_i(q_i) \quad (i = 1,\ 2) \tag{2}$$

Since p is a function of (q_1, q_2) and q_2 is an implicit function of q_1 and vise versa, the optimal solution should meet the condition:

$$MR_i(q_i) = \frac{\partial(pq_i)}{\partial q_i} = \left(\frac{\partial p}{\partial q_i} + \frac{\partial p}{\partial q_j}\frac{\partial q_j}{\partial q_i}\right) q_i + p = MC_i(q_i) = \frac{\mathrm{d}Cost_i}{\mathrm{d}q_i} \quad (i = 1,\ 2;\ j \neq i) \tag{3}$$

In the CV-based simulation method, *define the conjecture or belief of firm i on the response of its rival j to its change in production quantity as* CV_{ij}:

$$CV_{ij} = \frac{\partial q_j}{\partial q_i} \tag{4}$$

Substituting (4) into (3) yield:

$$\begin{cases} MR_1(q_1) = \left(\dfrac{\partial p}{\partial q_1} + \dfrac{\partial p}{\partial q_2}\ CV_{12}\right) q_1 + p = MC_1(q_1) = \dfrac{\mathrm{d}Cost_1(q_1)}{\mathrm{d}q_1} \\ MR_2(q_2) = \left(\dfrac{\partial p}{\partial q_2} + \dfrac{\partial p}{\partial q_1}\ CV_{21}\right) q_2 + p = MC_2(q_2) = \dfrac{\mathrm{d}Cost_2(q_2)}{\mathrm{d}q_2} \end{cases} \tag{5}$$

According to (5), (q_1, q_2) can be solved if estimations (or conjectures) on CV_{12} by firm 1 and CV_{21} by firm 2 are known. Here we assume that $p(q_1, q_2)$ is known by both firms based on historical data, while $Cost_i(q_i)$ is only known by firm i itself. Moreover, for a linear inverse demand function $p = e - fQ$ where $Q = q_1 + q_2$ and (e, f) are known from historical records, it is clear from (5) that firm i can make optimal decision q_i if and only if it can estimate $CV_{ij} (i \neq j)$ and product quantity of its rival $q_j (i \neq j)$ correctly; and that there is no need for firm i to know its rivals' cost functions.

3. Math model for CV based bidding in N-player spot market

In this section, the math model of CVBS is proposed for generation firms to improve their strategic behavior in real electricity spot markets.

Consider an N-firm spot market with inverse demand curve $p = e - fQ$, where p and Q are market clearing price and total generation respectively, e and f are known constants. Assuming that each generation firm has quadratic cost function and is rationally aiming at maximizing its profit, then the corresponding optimization problem for firm i ($i = 1, 2, \ldots, N$) can be defined as: