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A Lagrangian heuristic algorithm for a real-world train timetabling problem

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Abstract

The *train timetabling problem* (TTP) aims at determining an optimal timetable for a set of trains which does not violate track capacities and satisfies some operational constraints.

In this paper, we describe the design of a train timetabling system that takes into account several additional constraints that arise in real-world applications. In particular, we address the following issues:

- Manual block signaling for managing a train on a track segment between two consecutive stations.
- Station capacities, i.e., maximum number of trains that can be present in a station at the same time.
- Prescribed timetable for a subset of the trains, which is imposed when some of the trains are already scheduled on the railway line and additional trains are to be inserted.
- Maintenance operations that keep a track segment occupied for a given period.

We show how to incorporate these additional constraints into a mathematical model for a basic version of the problem, and into the resulting Lagrangian heuristic. Computational results on real-world instances from Rete Ferroviaria Italiana (RFI), the Italian railway infrastructure management company, are presented.

Keywords: Train scheduling; Path allocation; Lagrangian relaxation; Lagrangian heuristic; Computational results

1. Introduction

Management of main railway lines is increasingly becoming an important issue of European transport systems. Several reasons motivate better usage and planning of the rail infrastructure, particularly on the so-called European corridors, where track resource is limited due to greater traffic densities, and competitive pressure among the train operators is expected to increase in the near future. The availability of effective, computer-aided tools to improve the planning ability of railways over traditional methods is consequent to the new market scenario. Firstly, the re-organization of the European rail system, following the EU policy directives, has separated the activities of the Infrastructure Manager (who is responsible for train planning and real-time control) from the train operators (who provide rolling stock and

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transport services). This introduces the so-called access-to-infrastructure problem, in which several operators request capacity on a common railway line. Secondly, European railways are being transformed into more liberalized and privatized companies, which are expected to compete on a more profit-oriented basis. Thirdly, the rail transport system is subject to increasing pressure by governments and social interest groups to improve its overall efficiency and quality of service. Finally, the strategic character of the sector is highlighted in view of ecological impacts and national policies aiming at spilling freight traffic shares from roads to rails.

In this context, the ability to undertake infrastructure planning in a very timely, smooth and efficient way is becoming one of the most important tasks of the infrastructure manager, who at the same time has to optimize the use of the infrastructure and to provide track allocation, through rational and transparent procedures, as required in normative directives, introduced in the European Union since the early 90s.

The general aim was to implement scheduling algorithms which can provide a timetable plan on heavy-traffic, long-distance corridors. Through this model the infrastructure manager can allocate "optimally" the paths requested by all transport operators and proceed with the overall timetable design process, possibly with final local refinements and minor adjustments, as in the tradition of railway planners. The algorithm we present here is called traffic capacity management (TCM), and is part of a more general telematic architecture developed within the EU Project PARTNER. In brief, this allows each train operator to submit requests for paths on the given railway line, and allows the infrastructure manager to collect all the requests, run the optimization algorithm to allocate (if possible) all of them at maximum profit, and eventually respond to train operators with the proposed plan of track allocation and relative "access fees".

The essential characteristics of the process can be summarized as follows:

- train paths are given a value (i.e., a priority), an ideal timetable, with ideal departure and arrival times, and tolerances within which they can be "moved";
- the optimal allocation is found by maximizing the difference between the values of the trains scheduled and a cost penalty function, which takes into account the deviations from the ideal timetables;
- a subset of paths can be fixed and shall not be moved, being either prioritary (as is the case for long-term or clock-phased services) or already allocated (i.e., "sold" to some train operating company);
- the scheduling algorithm uses typical parameters as in the current timetable planning (e.g., minimum headway between trains, available tracks at each station);
- maintenance operations, also called "possessions", can impose constraints or forbid regular operations of the planned timetable during some intervals;
- signaling failures can cause some degraded operational mode, switching from an automatic to a manual block system.

In addition, under the assumption of competitive market, the process can be iterated if some operator does not accept the solution and asks for a re-evaluation by the infrastructure manager, e.g., by using modified path values, since the paths allocated take into account the value or access fee that the train operator is willing to pay.

The train timetabling problem has received considerable attention in the literature.

Many references consider mixed integer linear programming formulations in which the arrival and departure times are represented by continuous variables and there are logical (binary) variables expressing the order of the train departures from each station. Szpigel [14] considers a variant of these models in which the order of the train departures from a station is not represented by binary variables but by disjunctive constraints. The problem is then solved by branch-and-bound for small size instances by computing bounds through the relaxation of these disjunctive constraints. Jovanovic and Harker [8] solve a version of these models, that calls for a feasible schedule rather than for the optimization of a suitable objective function, by branch-and-bound techniques. Cai and Goh [2] illustrate a constructive greedy heuristic driven by one of these models. Carey and Lockwood [4] define a heuristic that considers the trains one at a time (in appropriate order), and for each train solves a mixed integer linear program analogous to those mentioned above in order to schedule the train optimally, keeping the path of the previously scheduled trains partially fixed. More precisely, the relative order of the train departures for these trains is kept fixed, whereas their arrival and departure times may be changed. Higgins et al. [7] define local search, tabu search, genetic and hybrid heuristics, finding a feasible solution by using a model in the family above.

Brännlund et al. [1] discretize the time into 1-min *time slots* and subdivide the track line into *blocks*. Operational constraints impose that two trains cannot be in the same block in the same time slot. There is a binary variable x_{sbj}